# Fundamental physical limits on computation: Summary of Results

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May 5, 1995

Here we succinctly summarize the main results of the author's paper "Fundamental physical limits on computation." We supress descriptions of the assumptions and the regimes of validity here, but, roughly speaking, my results are valid in the "high power limit" where the power, energy, or information density is reasonably large and also none of the dimensions of the container or window are very small, but nevertheless no mass-energy is so large that gravitational effects become significant. Bekenstein's results are supposed to be valid over an even larger regime than mine, including when gravitational effects are significant (but, they are weaker).

1B. Bound on sustained information flux $\dot{I}$ through a window of area $A$ when using power $P$:

$$\frac{\dot{I}}{A} \le \kappa_1 \, (\frac{P}{A})^{3/4}. \tag{1}$$

1C (Bekenstein). Transmission of a message in a time interval $t$ will require at least this much energy (or equivalent mass) to be transmitted:

$$\text{Energy per duration} - t - \text{bit} \quad \ge \quad \frac{\hbar \ln 2}{\pi t}. \tag{2}$$

2A. Bound on information $I$ contained in a region of volume $V$ containing gravitating mass $M$:

$$\frac{I}{V} \le \kappa_2 \, (\frac{M}{V})^{3/4} \tag{3}$$

2B (Bekenstein). Bound on information $I$ contained in a region enclosed in a surface of area $A$ and containing gravitating mass $M$ (need to assume "ideal gas conjecture"):

$$I < \kappa_6 \, M\sqrt{A}. \tag{4}$$

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3A (conjectural). Bound on rate of unavoidable decay of "maxent" information $I$ contained in a volume $V$. (Valid in the high-density limit $(I/V)^{1/3} r_c \gg 1$. In the low density limit, extremely slow decay rate is achievable.):

$$\frac{-d(I/V)}{dt} \ge \kappa_4 \, \ln(r_c[\frac{I}{V}]^{1/3}) \, (\frac{I}{V})^{4/3} \tag{5}$$

3B (conjectural; implied by 3A). Bound on the density of "stable" information. (Here $K$ is some dimensionless but unknown constant, presumably "of order 1" bit.):

$$\frac{I_{\text{stable}}}{V} < (r_c)^{-3} K \tag{6}$$

4D. Bound on the sustained computational speed $s$ (irreversible bit operations per second) of a computer enclosed in a surface of area $A$ and supplied with power $P$:

$$s \le \kappa_5 \, A^{1/4} P^{3/4}. \tag{7}$$

**Constants we've used above** [for the notation of the fundamental physical constants we use, see the full paper. $q$ is the number of "types" of particles in the universe; our results are pretty insensitive to $q$ since they depend on $(q/2)^{1/4}$ if at all. $r_c \approx 3.862 \times 10^{-13}$ meters is the Compton wavelength of an electron. All numerical values we give assume $(q/2) = 1$.]

$$\kappa_1 = 4\kappa_5/3 \approx 2.149 \times 10^{21} \, \frac{\text{sec}^{5/4}\text{bit}}{\text{Kg}^{3/4}\,\text{meter}^2}, \tag{8}$$

$$\kappa_5 = \frac{(\sigma_{SB})^{1/4}}{k_B \ln 2} \approx 1.612 \times 10^{21} \, \frac{\text{sec}^{5/4}\text{bit}}{\text{Kg}^{3/4}\,\text{meter}^2}, \tag{9}$$

$$\kappa_2 = (q/2)^{1/4} \frac{16}{3\ln 2} \frac{(\sigma_{SB})^{1/4} c^{5/4}}{k_B} \approx 3.392 \times 10^{32} \frac{\text{bit}}{\text{Kg}^{3/4}\,\text{meter}^{3/4}}, \tag{10}$$

$$\kappa_4 \approx \frac{45^{1/3} 5 (\ln 2)^{4/3} \alpha^2}{2^{2/3} 128 \pi^{8/3}} \frac{m_e^6 r_c^6}{\hbar^6} \approx 1.858 \times 10^{-58} \frac{\text{meter}}{\text{bit}^{1/3}\text{sec}}, \tag{11}$$

$$\kappa_6 = \frac{\sqrt{\pi} c}{\hbar \ln 2} \approx 7.267 \times 10^{42} \frac{\text{bit}}{\text{meter Kg}}. \tag{12}$$