Turing machine engineering and Immortality

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Abstract — **We address the either fundamentally important or silly question of whether the laws of physics permit a Turing machine (infinite computing device) to exist. "Can life can exist forever?" is a similar question. Although we do not settle the question, we propose a design which overcomes obstacles that at first would seem insuperable. It in fact is a Turing machine (and is an infinitely long-lived civilization) under certain restrictions. (Along the way we contribute several new cylindrically symmetric exact solutions of general relativity.)**

Keywords — Philosophy, general relativity, Turing machine, algorithmicity of physics.

1 Introduction

The topic of this paper is an either over compulsive or fundamentally important question: can a Turing machine exist? Equivalently "Do the laws of physics permit immortality?" We must explain what we mean by "immortality," "Turing machine," and "exist."

A "Universal Turing Machine" (or just "UTM") [21] is a simple kind of computer. The three important things about it are that (1) it has an infinite number of bits of read/write memory, (2) it can run for an infinite number of steps, (3) it can be programmed (by means of a program pre-written into a finite number of bits of the memory) to emulate any other reasonable kind of computing machine, in particular Turing's original machine. As far as we are concerned any machine with these 3 properties will do, and its details are not important. (Indeed for us "cellular automata" [21] seem to be a more useful UTM model.) Optionally, we shall also permit "nondeterminism," i.e. giving our UTM access to sources of random bits – this enhances its capabilities. UTMs are the most important concept in computer science. A UTM running for infinite time could, e.g., compute all the digits of π, decide whether the Riemann hypothesis (one of the greatest unsolved problems of mathematics) is true, or simulate an entire civilization forever.

Viewing "life," or intelligent life anyhow, as a "computer," the question of whether "immortality" is possible may be viewed as the question of whether a computer can exist that can perform an infinite number of steps – and if we want our computer not merely to return to previous states and execute an infinite loop, it must have an infinite memory.

Presumably, if the universe is finite, then a Turing machine cannot exist. It is not known if our universe is finite, but the present observational evidence and theoretical models predict that it will continue to expand at a rate sufficient that our local group of galaxies will ultimately lose touch with the rest of the universe, while meanwhile all the stars burn out. Thus in another 10^{10}-10^{11} years, it is expected that our universe will become very cold, isolated, and dark. The recent claim that there is a repulsive cosmical constant causing the universe's expansion to be exponentially *accelerating* seems to kill all hope for any interesting kind of infinite machine in our universe[1].

[1] Krauss & Starkman [17] argue that, in any eternally expanding universe, "the integrated conscious lifetime... [and] the total information recoverable by any civilization over the entire history of the universe is finite" and hence that "life cannot be eternal." Although I think their claims are essentially true, I do *not* agree that they have a *proof* – they merely have refutations of a nonexhaustive set of design ideas. They are also vulnerable to the following 4 criticisms. (1) They seem to have the idea that a "living creature" is a finite state machine, and they are interested in whether such a creature can live forever. I would argue this is a silly question; the interesting question is whether it can live for N timesteps where N is the number of states ("effective immortality"). The arguments of [17] may easily be strengthened to suggest that N can easily be made large enough, e.g. $N = 10^{10^{15}}$, so that any creature will die far before N timesteps. (2) Their arguments assume a homogeneous cosmical model of the Friedmann Robertson Walker (with cosmical constant) type. They thus ignore the possibility that inhomogeneities could be exploited by life in its quest for survival. (Indeed, the present paper employs a very inhomogeneous universe.) (3) It may be possible for life to tap the expansion of the universe, and/or the cosmical constant, as a source of power. (4) Bennett [1] pointed out that a finite state machine could be designed to use *no* power. It would run off of random thermal fluctuations in a medium such as room temperature water and would, by "Brownian motion" make FSM transitions either in the forward or backward time direction. After N^2 random transitions it becomes likely that N more forward

But we are not interested in such trivial objections. What we shall mean by "exist" is that, using the laws of physics as they are currently understood – but perhaps in a different and "better" universe, which is infinite, has all the matter initially placed precisely where we want it (subject to precision limits from quantum uncertainty principles), and has an unlimited power source available – could a Turing machine exist? In other words: *given the same laws of physics, but different initial conditions, could we get a UTM?*

There are three different sources of the difficulty of attacking this problem:

1. We do not know what the laws of physics are.
2. Many of the laws that have been proposed are presently beyond the reach of rigorous mathematical analysis[2].
3. How can we handle the infinities in the definition of Turing machine?

In this paper we are mostly going to dodge the first two difficulties and concentrate on the third. We shall assume general relativity works and shall avoid using it in regimes (e.g. involving singularities) where it is suspect; and all the matter and radiation we shall employ will be of the ordinary sort at earthlike temperatures, energies, pressures, and densities where we understand it the best. We shall in general be optimistic about questions of whether little understood exotic physics such as GUTs, quantum gravity, dark matter, extremely rare enormous macroscopic quantum effects, extremely rare enormous thermal fluctuations, etc., could step in to ensure the destruction of our creations.

The question of whether the laws of physics permit a UTM seems of fundamental and central importance for computer science, philosophy, and physics. It is relevant to the key question of how computationally complicated it is to understand and use theoretical physics. E.g., if a Turing machine is possible, then it is a standard immediate consequence [21] that there exist undecidable long-term physical prediction problems which have definite yes/no answers fully determined by the fully known initial data and the fully known laws of physics – *but* those answers cannot be determined by any algorithm. If a computing machine with super-Turing power is possible, then even worse pathologies can happen, e.g. finite-time algorithmic unpredictability of physics. I had previously considered the algorithmicity status of subsets of approximate physical laws, e.g. just Newton's classical laws of motion [27], just the Navier-Stokes equations of hydrodynamics [29], just the nonrelativistic Schrödinger equation quantum N-body problem [28], etc. (The conclusions were, essentially, that the latter is algorithmically simulable for any finite time to any desired precision, but the former two are not.) However, this is the first attempt to look at *all* of the *exact* laws of physics.

On the other hand it could also be viewed as silly, e.g. since, in any finite amount of future history, humanity is never going to build any infinite object. To that we riposte that the question of whether humankind (and/or its descendants, or any other species of life) *itself* could exist forever, either in this, or any other universe, is essentially the same question.

2 Initial naive thoughts and their failure

One's naive immediate reaction to this question is "yes, it is obvious that a UTM could exist." After all, computers have been built. All we need to do is postulate an infinite number of them, or just one with access to an unbounded supply of read/write tape. There was a previous analysis of our question by Michael P. Frank in his PhD Thesis [11], which indeed concluded UTMs should be possible.

But after thinking some more and proposing some rough designs, we soon discover (we will do this below) that they don't work. In particular, we'll soon argue that Frank's ideas ultimately won't work. Soon pessimism sets in and one conjectures that UTMs are obviously impossible!

After a long fight, one reaches the third mental step, which begins in the next section of paper. Starting there, we will present a design arguing that a UTM is possible. However, in the final section we shall raise unanswered questions about it which suggest that this design still fails. Even if so, though, it does accomplish *something* interesting.

Frank observed (as was also mentioned by Gacs [12] and probably many others) that a ball-shaped computer of radius r would generate heat power proportional to r^3, whereas the heat could only be removed (at any bounded temperature) at a rate proportional to r^2. So, ultimately, this design would fail. (Indeed, a typical present day

transitions have been made than backward transitions and the computation is complete. (By using K independent copies of the machine, such victory happens with probability exponentially close to 1 for K large.) Bennett [2][3][19] also demonstrated that a reversible Turing machine has the same computational power as an irreversible one. Since Krauss and Starkman argue that, in a De Sitter expanding universe, a constant nonzero temperature will be asymptotically approached, we conclude the *possibility*, not the impossibility, of this kind of immortal "living creature"! Although it might be counterargued that decay processes would prevent that, Krauss and Starkman never made such an counterargument, and furthermore, I believe that decay could be delayed exponentially long, permitting "effective immortality" for this kind of creature.

[2] For example, the eternal existence of solutions to general relativity has not been proven. Quantum field theories are in an even worse state.

personal computer emits over 100 times more heat per unit volume than the sun.) Therefore, Frank proposed, our computer should be an infinite 2-dimensional sheet, with power supplied, and heat removed, along the 3rd dimension. This is, in fact, close to the way present day computer chips are built.

But this design also, ultimately, fails. If the sheet has some constant areal mass density, then the mass in a disk of radius r is proportional to r^2. But the Schwarzschild radius [22][31] of a mass M is *linearly* proportional to M. Therefore, any sufficiently large disk of our sheet, would, in fact, collapse into a black hole. Put another way, no static spacetime metric exists[3] corresponding to an infinite homogeneous plane sheet of mass. This highlights the fact that the branch of physics most relevant to questions about very large (and infinite!) objects is *General Relativity* [15][22][31].

This suggests that, to avoid collapse, our UTM design should have mass M that grows *at most linearly* with the radius r of any ball. So consider an infinitely long rod. Model it as an infinite collection of equal point masses at all the integer points of the real line. Then the Newtonian gravitational attractive force between the left half-rod and the right half-rod is proportional to

$$\sum_{p\geq 1}\sum_{q\geq 0}\frac{1}{(p+q)^2}=\infty \tag{1}$$

so that any length-1 chunk of our rod would have to withstand infinite compressive stress. I.e., this design idea (at least to the extent Newtonian gravity is valid) fails.

There is also another problem with it. Suppose our rod emits a constant amount of thermal radiation per unit length (to get rid of waste heat, or merely because it is at some constant nonzero temperature). Then a ball of radius r centered on our rod, would, ultimately for large r after a long time, contain thermal energy proportional to r^2. Since the Schwarzschild radius of a mass M is linearly proportional to M, while r^2 is superlinear, this suggests that the *waste heat* from our computer would ultimately collapse it into a black hole!

This waste-heat obstacle suggests that we must design our computer to have mass M that grows *sub*linearly with r. So suppose that our design consists of an infinite number of essentially identical ball-shaped computers flying through space and communicating with each other via laser beams and telescopes. Suppose each one of these computers emits heat at a constant rate. Then the large sphere of radius r surrounding that particular computer will (after sufficient time) ultimately contain heat energy proportional to r. This linear growth is permissible as far as the Schwarzschild radius limitation is concerned, *but* any superlinear growth, even $r \log\log\log r$ growth, would be impermissible. That forces each ball to be receding from its nearest neighbor with at least constant speed. But if so, then after some finite time, our computers will lose the ability to communicate with their nearest neighbors, and hence with any other computer, so that our UTM again will self-destruct. (E.g.: if our computer emits signal photons at some constant rate, then the neighboring computer distance s away will receive a number of photons per unit time proportional to s^{-2} , so that if s is growing linearly with time, the total number of photons *ever* received will be *bounded* since $\int_1^\infty s^{-2}\mathrm{d}s<\infty$.) Again, failure.

Yet another problem we have to overcome is that, due to thermal noise at any fixed nonzero temperature, at least some of our transmitted bits will be corrupted. But we need our Turing machine to perform an infinite number of bit-operations without a single failure!

Still another problem: All known materials at fixed nonzero temperature evaporate in vacuum. So eventually our whole computer would evaporate. Similarly, planetary atmospheres gradually escape because some of their molecules, in the high tail of the Maxwell-Boltzmann velocity distribution, occasionally via thermal effects exceed escape velocity. For us, this escape velocity seems necessarily sublight because otherwise our constructs would be "black holes" which waste heat could not escape from. Thus we expect everything necessarily will evaporate.

Even assuming the evaporation problem is somehow overcome, there is a more severe and fundamental-sounding problem: atoms decay. Namely, Grand Unified Theories of particle physics predict that the proton, and hence all atoms, are radioactive. Protons should decay into leptons (e.g. positrons) mesons, and photons – and the mesons then would decay into photons; and the positrons would annihilate with nearby electrons, with the net result being a decay of a hydrogen atom into pure energy. It seems difficult to build a Turing machine if all the atoms in the universe are going to disappear.

So far, no experimental evidence supports proton decay – the proton lifetime is known to exceed 10^{33} years and no instance of baryon number nonconservation has been seen. Nevertheless, I argue that it *is* clear that atoms decay, even though it is unclear whether GUTs are correct. Why? Because all theoretical physicists believe in Hawking

[3] I do not know if this nonexistence statement has been proven, but it is easy to prove that a static metric representing a radius-r constant-thickness *spherical shell* of matter whose density is bounded between two positive constants and spherically symmetric, cannot exist for any sufficiently large r. This follows from Birkhoff's theorem and the Hawking-Penrose trapped-surface singularity theorems [15]. This is essentially what we wanted to prove, provided one is willing to regard a "homogeneous plane sheet" as the $r \to \infty$ limit of such a spherical shell. For a proof of a generalization of the Birkhoff and Taub uniqueness theorems in general relativity, see [18]. The latter theorem is that the only vacuum GR solutions that are plane-symmetric (or even conformally related to a metric with plane symmetry) are the Taub and Kasner metrics $\mathrm{d}s^2=\pm z^{-1/2}(\mathrm{d}z^2-\mathrm{d}t^2)+z(\mathrm{d}x^2+\mathrm{d}y^2)$. (Only the former of these is static.) Observe that both of these are only defined in a *half* space and are singular at $z=0$.

radiation from black holes. The existence of Hawking radiation proves that a quantum-gravitational mechanism exists to allow atoms to convert into particles of lower mass and zero baryon number. This, it seems to me, *proves* that the proton, and hence all atoms, are radioactive with some (perhaps extremely long) halflife. (I have not seen this remark before.) Assuming, extremely crudely, that the decay time is proportional to the inverse square of coupling constant, leads to a proton halflife estimate of $\approx 10^{115}$ years.

The net effect of all of these subproblems combined is not a proof that UTMs are impossible (it is merely a set of refutations of a nonexhaustive set of possible design ideas) but it certainly is enough to make us very pessimistic.

3 A Universal Turing Machine design

Although at first all of the above obstacles appear insuperable, surprisingly, it appears possible to overcome all of them! A major insight is that General Relativity can behave quite differently from its approximation, Newtonian gravity.

There are 5 main ingredients of our design:

1. General relativity permits a static spacetime involving an infinite rod to exist with only bounded and finite stresses and density everywhere, and with no metrical singularity anywhere.

2. General relativity permits power (in the form of electromagnetic radiation) to flow radially inward from infinity to our rod, and heat (thermal radiation) to flow radially outward from our rod to infinity, both at constant rates per unit length, again in a static spacectime metric with no singularity ever. (Note, we assume these power flows as "boundary conditions at infinity.")

3. Although light can escape from our rod to infinity, it is infinitely redshifted as it does so. In other words, the escape velocity is precisely lightspeed *but* there is no "black hole." Therefore, the atmosphere of our rod "planet" would not escape and we would not suffer evaporation (or at least, any such evaporation would be in permanent equilibrium with recondensation).

4. The ultraslow decay of atoms into energy may be compensated for by synthesizing nucleons in accelerators.

5. A hierarchical error correction scheme by Peter Gacs [12] permits a 1-dimensional Turing-universal cellular automaton to operate even in the presence of random noise which corrupts its cell states (at some sufficiently small constant rate of state-corruption per cell per timestep) and still (with success probability bounded above zero) complete its programmed *infinite* computation without ever experiencing even a single uncorrected error.

We soon shall discuss each of these in more detail. It then is not particularly hard (though we shall not discuss the details) to festoon our rod with a constant number of identical computers per unit length, each communicating with a constant number of its nearest neighbors, and each optionally equipped with true random bit generators based on quantum effects. Alternatively, we could regard our rod as an infinite-size "planet" on whose surface humans, or other lifeforms, could live forever. (Due to the cylindrical symmetry, the surface gravity would always pull radially inward, i.e. perpendicularly to the rod's surface.)

3.1 A menagerie of metrics

We now list[4] some static cylindrically symmetric spacetime metric solutions of general relativity. In all cases we shall use coordinates t, r, z, θ with $r \geq 0$ and θ taken modulo 2π, and the cylindrical symmetry and staticity of each metric are readily checked by verifying its invariance under t, z, and θ translations. We shall use units with lightspeed$= c = 1$ throughout. We shall assume basic knowledge [22][31] of Einstein's field equations $G_{\alpha\beta} = 8\pi G_{\text{Newton}} T_{\alpha\beta}$ where $G_{\alpha\beta} = R_{\alpha\beta} - R g_{\alpha\beta}/2$ is the Einstein tensor (here defined in terms of the Ricci curvature tensor $R_{\alpha\beta}$, its contraction $R = R^{\mu}_{\mu}$, and the metric tensor $g_{\alpha\beta}$), $T_{\alpha\beta}$ is the energy-momentum tensor of the matter and/or radiation in the universe, and $G_{\text{Newton}} \approx 6.67 \times 10^{-11}$meter^3/kg/second^2 is Newton's gravitational constant. For all the metrics we shall give here, the energy density ρ and the principal pressures P_r, P_θ, and P_z (all three of which are equal for an "isotropic" metric) are: $\rho = -T^t_t$, $P_r = T^r_r$, $P_\theta = T^\theta_\theta$, and $P_z = T^z_z$ (in which, of course, Einstein summation convention is not intended).

All the metrics we shall list are "twist-free." All are asymptotically extremely nonflat for large r. This distortion will never be enough to create a singularity or event horizon – our metrics will plainly be nonsingular for all $r > 0$ – but it will be enough to create infinite redshift, i.e. time distortion. That is, light signals sent from a static observer

[4]Although we shall not use them, it also is worth noting the stunning *non*static never-singular cylindrically symmetric cosmologies of Senovilla et al. [7] (filled with isotropic thermal radiation) and Mars [20] (filled with stiff perfect fluid), and Senovilla & Vera's also-remarkable dust-filled cylindrical cosmology, featuring naked singularities [26].

at $r = r_1$ to a static observer at $r = r_2 > r_1$ are redshifted; and as $r_2 \to \infty$ by an infinite factor. That claim may be verified simply by noting that g_{tt} is a function of r which varies by an infinite factor as $r \to \infty$.

Many of the metrics we list are new, some are rediscoveries albeit expressed in different coordinate systems so that it is very nonobvious whether they are really rediscoveries, and some others correct errors in previous papers. The net effect of our list will be a substantial increase in knowledge about static twist-free cylindrically symmetric spacetimes.

1. A 1-parameter family of metrics attributed [5] to T.Levi-Civita in 1919 is

$$\mathrm{d}s^2 = -r^{4m}\mathrm{d}t^2 + r^{(8m-4)m}(\mathrm{d}r^2 + \mathrm{d}z^2) + r^{2-4m}\mathrm{d}\theta^2. \tag{2}$$

This is all cylindrically symmetric solutions of Einstein's *vacuum* field equations. If $m = 0$ or $m = 1/2$ this is just flat space, albeit in the latter case in a peculiar coordinate system (the flatness may be verified by computing $R^{\alpha}{}_{\beta\mu\nu}$ and seeing that it is 0). If $m \notin \{0, 1/2\}$ then this metric is singular when $r = 0$ only (as $r \to 0$, $R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} \to \infty$), and since there is no event horizon, this singularity is "naked." If the constant m has small absolute value, then [24] there is some rationale for regarding it as the "mass per unit length" of the line singularity on the symmetry axis. In general, though, the notion of "mass per unit length" in general relativistic cylindrical systems is meaningless [24][4]. Philbin [24] explicitly showed[5] that the Levi-Civita metric could be regarded as the gravitational field of a cylindrical perfect fluid source if $-1/2 < m < 1/2$.

The gravitational attraction is inward everywhere if $m > 0$ but repulsive everywhere if $m < 0$. If $m > 1/4$ then no circular orbits (timelike circular geodesics) exist. If $m = 1/4$ then all axial circles are null, i.e., are "photon orbits." In the most physically interesting range $0 < m < 1/4$ every circle is a sublightspeed orbit.

2. The new metric

$$\mathrm{d}s^2 = \exp(Kr^2)(\mathrm{d}t^2 - \mathrm{d}r^2) - \mathrm{d}z^2 - r^2\mathrm{d}\theta^2 \tag{3}$$

represents a space completely filled with "stiff fluid," i.e. perfect fluid obeying the relation $\rho = P$ where P is its pressure and ρ its density. (Such a fluid would have speed of sound equal to lightspeed, i.e. would be as stiff as it could possibly be.) That density is $\rho = (8\pi G_{\mathrm{Newton}})^{-1} K \exp(-Kr^2)$. It is nowhere singular.

3. The (also new, also nowhere singular) metric

$$\mathrm{d}s^2 = e^{Kr}(\mathrm{d}t^2 - \mathrm{d}r^2) - \mathrm{d}z^2 - r^2\mathrm{d}\theta^2 \tag{4}$$

has these as the nonzero components of its mixed Einstein tensor

$$-G^t_t = G^r_r = \frac{K}{2re^{Kr}}, \tag{5}$$

while for its covariant Einstein tensor

$$G_{tt} = G_{rr} = \frac{K}{2r}. \tag{6}$$

Hence according to Einstein's field equations, this represents vacuum filled with bidirectional null radiation traveling in the inward and outward r directions only, with equal intensities. For the energy density of this radiation to be positive, K must be positive.

It is possible to regard the incoming radiation as coherent radio waves obeying Maxwell's equations, but if so, then the outgoing radiation cannot also be regarded both as obeying Maxwell's equations and as exactly cylindrically symmetric[6]. Instead it may be regarded as incoherent radiation – a good approximation provided its wavelength is substantially shorter than $1/K$.

4. Evans [10] found the following metric

$$\mathrm{d}s^2 = -(1+x^2)^{2/3}\mathrm{d}t^2 + \mathrm{d}r^2 + (1+x^2)^{-1/3}(\mathrm{d}z^2 + r^2\mathrm{d}\theta^2) \tag{7}$$

where $x = Kr$ and K is a constant. This represents a space filled with isotropic fluid obeying $\rho = 5P = 5(24\pi G_{\mathrm{Newton}})^{-1}K^2/(1+x^2)^2$.

5. The following (new) metric

$$\mathrm{d}s^2 = e^{F(r)}(-\mathrm{d}t^2 + \mathrm{d}r^2) + e^{2J(r)}(\mathrm{d}z^2 + \mathrm{d}\theta^2) \tag{8}$$

represents a cylindrically symmetric static spacetime filled with radiation in thermal equilibrium, if $F'(r)$ and $J'(r)$ obey

$$F' = -(J'' + 3J'^2)/J' \tag{9}$$

[5] A previous demonstration of this for $0 \le m < 1/2$ by Bonnor and Davidson [5] contained errors at the core of its argument: namely it was claimed incorrectly that there is zero pressure at $r = r_1 > 0$ for the associated metric discussed in their section 2, whereas in fact, its pressure is positive for all $r \ge 0$. (We shall discuss that metric later.) They gave wrong formulas for its pressure and density. Therefore, all the interpretational results in [5] about the Levi-Civita metric would have to be regarded with suspicion, were it not for Philbin's work [24].

[6] This may be shown by computing the Rainich-Misner-Wheeler conditions.

and

$$J'J''' - J''J'^2 - J''^2 - 6J'^4 = 0. \tag{10}$$

(These cause the Ricci curvature scalar to be $R = 0$ and cause isotropicity: $G_r^r = G_z^z = G_\theta^\theta$. That and the fact that G_α^β is diagonal validate our interpretation of the metric as a photon gas.) The solution of the latter differential equation is

$$\int^{J'} \frac{u^{-2}\mathrm{d}u}{3 + \mathrm{RootOf}(Z^5 + 10Z^4 + 25Z^3 - C_1 u^{-5})} = C_2 + r. \tag{11}$$

If we agree to take $C_1 > 0$ and $u > 0$ then the quintic has a unique positive real root. If we use it, then $J'(r)$ plainly monotonically increases with r. If we agree to take $C_2 \ge 0$, then plainly $J'(r)$ is positive everywhere r is. Then $J(r)$ is monotonic increasing and concave-∪ for all $r \ge 0$, causing $F'(r)$ to be negative for all $r \ge 0$ so that $F(r)$ is monotonic decreasing. The metric is then evidently never singular for $r > 0$, and if we make J and F behave right at $r = 0$, there is no singularity there either.

A static cylindrical metric representing radiation in thermal equilibrium was also found by Teixeira, Wolk, and Som [30]:

$$\mathrm{d}s^2 = -F^3\mathrm{d}t^2 + F\mathrm{d}r^2 + \frac{1}{F}(\mathrm{d}z^2 + r^2\mathrm{d}\theta^2) \tag{12}$$

where $F = (1 + r^2K^2)^{2/5}$ and K is a constant. The pressure is

$$P = (8\pi G_{\mathrm{Newton}})^{-1}\frac{(4/5)K^2}{(1 + r^2K^2)^{12/5}} \tag{13}$$

and $\rho = 3P$. This presumably is equivalent under some coordinate change to our metric, but it is in most ways much nicer to handle.

It is also possible (and in many ways) to interpret the radiation as *non*thermal and nonequilibrated. Regard thermal radiation as divided into different kinds of photons. Suppose two red photons have the same energy as a blue photon. If we remove some subset of blue photons from isotropic thermal radiation and replace each of them with two red photons, then the metric will be unaffected.

6. The following 3-parameter family of metrics was found by Davidson [8] (as restated in streamlined form in EQ 2.1 of [5])

$$\mathrm{d}s^2 = -F^{2E-4K}\mathrm{d}t^2 + F^{2E}\frac{B^{-2}\mathrm{d}r^2}{\overline{F}^2 - 4F} + F^{2K}(\overline{F}^2 - 4F)\mathrm{d}z^2 + \frac{C^2}{B^2}F^{2K}r^2\mathrm{d}\theta^2 \tag{14}$$

where $F(r) = A^2 + r^2$, and 4 constants are defined by $\overline{F} = A^2 + \overline{r}^2$, $E = (4 - A)/(A^2 - 4)$, $K = -1/(A + 2)$, and $C = A^{2K-2E}/\sqrt{\overline{F}^2 - 4A^2}$ in terms of the three parameters $\overline{r}$, A, and B. I have confirmed (by computing G_α^β) that this indeed represents a space filled with a isotropic fluid. However, Davidson [8] gave incorrect formulae for the pressure, density, speed of sound, and equation of state of this fluid. Those errors then contaminated his successor paper [5] (e.g. EQ 2.4 and 2.5 of [5] are incorrect). Possibly the source of the error was the fact that they quote an incorrect version of the Einstein field equations (as EQ 2.6 of [5]), or that they used G_{rr} or G^{rr} when computing the radial presure, as opposed to the[7] correct G_r^r (e.g. see the misleading EQ 2.3 to 2.6 in [10], which [8] built upon), or possibly it was algebraic manipulation mistakes[8]. The correct formulae are

$$P = (r^2 + A^2)^{-2(A-1)A/(A^2-4)}Q, \quad \rho = (2A - 3)P \tag{15}$$

where the constant Q is

$$Q = \frac{B^2A^2}{2\pi G_{\mathrm{Newton}}}\frac{(\overline{r}^2 + A^2)^2 - 4}{A^2 - 4}. \tag{16}$$

If $A > 2$ then $0 < P < \rho$ and $P'(r) < 0$ and $\rho'(r) < 0$ for all $r \ge 0$. The pressure P is *not* 0 at $r = \overline{r}$ (nor at any $r \ge 0$) contrary[9] to claims by Davidson [8][5].

This is actually a much simpler equation of state than the incorrect one originally proposed, and the speed of sound $1/\sqrt{2A - 3}$ is constant. A presumably equivalent (since the fluid has the same equation of state and the solution has the same qualitative properties) metric was then found by Kramer [16] 6 years later. But well before Davidson, Bronnikov [6] had already found a solution equivalent to Kramer's.

Presumably the special case $A = 3$ is equivalent to our EQ 8 and the Teixeira photon gas metric EQ 12.

[7] Only the *mixed* Einstein tensor here yields invariance of the pressure and density under coordinate rescalings.

[8] All the metrics in the present paper have been checked with the computer software system `GRtensorII`.

[9] Davidson erroneously thought his metrics represented rods with a zero-pressure outer surface at $r = \overline{r}$; all the quantities with bars (e.g. $\overline{F}$) denote the corresponding unbarred quantities at $r = \overline{r}$.

7. Similarly, Davidson claimed that the special case $A = 4$ of his metric had previously been discovered by Evans [10] in a different coordinate system – but in fact, the form Evans gave[10] for this metric is incorrect since it is not isotropic (G^θ_θ, G^r_r, and G^z_z are not all equal). Fortunately, the metric that Evans must have *meant* was rediscovered by Kramer [16] and Haggag & Desokey [14]. It is

$$\mathrm{d}s^2 = -e^{4r}\mathrm{d}t^2 - \frac{27Ye^{6r}}{4\sinh(3r)\sinh(3r-T)}\mathrm{d}r^2 \tag{17}$$
$$-e^r\sinh(3r-T)\mathrm{d}z^2 + Y\sqrt{1+A^2-5A/2}\sinh(3r)e^r\mathrm{d}\theta^2$$

where

$$T = \frac{1}{2}\ln\frac{2A-4}{2A-1} \tag{18}$$

and Y and A all are constants, with $0 < A < 1/2$. This represents a rod of material obeying the equation of state $\rho = \overline{\rho} + 5P$ where $8\pi G_{\mathrm{Newton}}\overline{\rho} = (4/3)Y^{-1}\sqrt{(4A-2)/(A-2)}$. The zero-pressure surface of the rod is at $r = \overline{r}$ where $12\overline{r} = \ln([2-A]/[2-4A])$. This solution has $0 < P < \rho$ and $P'(r) < 0$ for all r with $0 \le r < \overline{r}$. Since Davidson's *corrected* equation of state is not the same as the equation of state here, it is *not* true (contrary to Davidson's claim) that this is a special case of EQ 14.

8. Haggag & Desokey [14] and Philbin [24] gave 5 different families of metrics representing the interiors of finite-radius cylindrical rods made of different kinds of matter. All of these solutions obey $P(\overline{r}) = 0$ and $0 < P < \rho$ and $P'(r) < 0$ and $\rho'(r) < 0$ for all r with $0 \le r < \overline{r}$.

9. Perhaps the model of a rod corresponding most closely to people's usual understanding of the word "rod," would combine *constant* density with isotropic pressure (an "incompressible fluid"). That would be the cylindrical analogue of the spherically symmetric "Schwarzschild interior solution" [31]. It was noted in 1996 [14] that it was "remarkable that, after 80 years of work... [this] problem... remains unsolved." We'll now see that it indeed is possible to find such a metric, but unfortunately it seems to be complicated. We sketch the procedure to find it. We begin with the metric of EQ 8 involving two undetermined functions $F(r)$ and $J(r)$. We then demand of $F(r)$ and $J(r)$ that $G^t_t = -K$ for some positive constant K (i.e. we demand positive constant density) and $G^r_r = G^z_z$ (isotropicity; the other isotropicity condition $G^\theta_\theta = G^z_z$ is satisfied automatically by this metric). This yields

$$F'J' - 2J'' - 3J'^2 = Ke^F \tag{19}$$

and

$$2F'J' = F'' + 2J''. \tag{20}$$

Because the latter ODE is first order linear in J', it may be solved by "the method of the integrating factor"

$$J'(r) = \frac{1}{2}e^{F(r)}\int^r F''(u)e^{-F(u)}\mathrm{d}u \tag{21}$$

to reduce the problem to a *single* integro-differential equation (EQ 19 with EQ 21 substituted in) to be satisfied by $F(r)$.

3.2 Combined metric

We may glue any interior metric from the above Chinese menu to any exterior one at a common cylindrical surface $r =$ constant, yielding a combined metric. The interior and exterior cylindrical surfaces merely need to have equal circumferences and equal radial pressures.

Let us choose EQ 8 (thermal radiation exterior) or the presumably equivalent $A = 3$ special case of EQ 14, or the presumably equivalent metric of EQ 12 as our exterior.

As our interior, choose either the constant-density rod, or the "Evans" metric EQ 17, or any of the Philbin [24] or Haggag-Desokey [14] metrics. Of these, EQ 17 seems simplest. Note that by making P *small* in comparison to ρ everywhere within our rod we get a material of near-constant density, similar to common materials in everyday life. For example, a chunk of iron at the Earth's surface has density 7.8gram/cm^3, which, multiplied by c^2 to express it in the same units as pressure, is 7×10^{20} pascals. Meanwhile our iron's pressure is of order 1atmosphere$\approx 10^5$ pascals, i.e. 7×10^{15} times smaller than its density. Even in the Earth's center, the pressure is only about 3×10^{11} pascals, i.e. still over a billion times smaller than the density. The speed of sound in iron is 5130 meters/second, i.e. 0.000017 in units with $c = 1$, so that $A = 1.7 \times 10^9$ would seem roughly appropriate, at least for the purpose of matching the sound speed, if the rod material were Davidson's metric EQ 14. (Although this metric has no zero-pressure surface and hence cannot be regarded as an interior for the Levi-Civita *vacuum* metric, it does have surfaces with arbitrarily

[10] Defined by [10] as the physical case of his "model ii" in his EQs 2.1, 2.10, 2.11, and 3.2, and in the discussion directly before EQ 3.2.

small pressures and hence could perfectly well be matched to our *non*-vacuum exteriors.) The object of choosing parameters such as Davidson's A is to make the pressure at the surface of our rod be small enough to match the pressure from the radiation exterior, despite the fact that the density of the rod will be enormous compared to the equivalent mass-density of the radiation.

Although no homogenous material *exactly* obeys an equation of state of the form $\rho = \text{const}$ (nor, probably, even $\rho = 5P + \overline{\rho}$ as in EQ 17) we could make a rod whose composition varies with radius. Such a rod *could* exactly obey the $\rho(r)$ and $P(r)$ curves given by either of these exact solutions. This is a standard remark which we repeat here to make it clear that our combined interior-exterior metric is an exact solution of general relativity representing a genuine physical scenario.

Our combination metric has fascinating properties. It may be regarded as an infinitely long rod surrounded by a photon-filled vacuum. Astonishly, this rod remains unchanging with time, even if it is made of inviscid *liquid*[11]; the stress inside it is purely in the form of bounded isotropic hydrodynamic pressure. The material of the rod has entirely realistic properties.

Its exterior may be regarded as a vacuum containing incoming electromagnetic radiation mixed with an equal amount (in terms of energy) of outgoing radiation, so that the net energy flux is everywhere isotropic. The incoming radiation could be absorbed by solar cells on the rod's surface, used to power computers or biomass distributed within the rod, and then reradiated as heat (same amount of power, but in the form of redder radiation).

The incoming radiation is not interfered with by the outgoing waste heat radiation because Maxwell's equations obey the principle of linear superposition. However, there is a quantum electrodynamic effect called "photon-photon scattering" [9] which, although small at low frequencies f (the cross section is proportional to f^6), causes nonzero nonlinear effects. It is easy to confirm that the intensity of our radiation falls off as a sufficiently large power of radial distance so that the typical incoming or outgoing photon will scatter off only a small constant expected number (far below 1) of other photons in its entire infinite history.

The velocity required to escape from the surface of the rod (or from anywhere else) to infinity is lightspeed, a fact revealed by the infinite redshift.

3.3 Nucleon synthesis

Assume there is some kind of unknown high energy physics (GUTs? Quantum gravity?) that allows atoms to decay into energy. (If atoms are stable, all this is not an issue.) Then that physics presumably also allows atoms to be created from energy and presumably that physics, operating in the early universe, was responsible for creating all the hydrogen in the universe. Furthermore, since the universe is made of matter and not antimatter, this must happen in such a way as to slightly favor the production of hydrogen versus antihydrogen; and since the early universe only lasted a very short time, evidently that synthesis can be accomplished very quickly. So, our proposal to overcome the ultraslow decay of atoms such as hydrogen into energy is simply to synthesize replacement atoms in particle accelerators. Particles often are synthesized from pure energy in accelerators, but so far this has only been done in a way respecting the conservation of baryon and lepton number. We need to violate baryon and lepton number conservation (as does any process which allows atoms to decay into energy).

What theory is known is summarized in [25] pages 85-88, 115-130, and 141-156. To explain the matter/antimatter excess of the universe, it suffices to have

1. baryon and lepton number nonconservation,
2. CP violation, and
3. thermal nonequilibrium in the hot early universe.

But no subset of these 3 ingredients suffices. Baryon and lepton number are exactly conserved in the "standard model" and hence physics beyond the standard model must exist[12]. It has been estimated that (1) will occur at "GUT scale"or "SUSY scale" particle energies exceeding 10^{14}GeV= 16kJoule (which is about 300 times larger than the most energetic cosmic rays seen so far) although possibly energies as large as the "Planck scale" 10^{19}GeV= 1.6GJoule are required. It is known that (2) happens within the standard model, although it is currently believed [23] that this effect is numerically too small by factor of 10^9 to explain the current state of the universe, in which case again, physics beyond the standard model is required. Conditions of thermal nonequilibrium were thought to be achieved in the early universe because of rapid early expansion, and are trivial to achieve in particle accelerators.

Consider this crude calculation. The Earth contains about 4×10^{51} "nonradioactive" nucleons. Assume they actually decay into energy with mean life of 10^{35} years (the currently most popular guess). Then the Earth's total

[11] A design involving an anisotropically stressed solid could be criticized since atomic diffusion could ultimately allow strains to relax. Our design is not subject to such a criticism.

[12] Other known inadequacies of the standard model include the fact that neutrinos have mass, and the fact that about 80% of the gravitating matter in the universe is "dark matter" made of something other than the standard particles.

rate of energy loss from this effect is then about 5 megawatts. Assuming the synthesis of replacement matter has efficiency 10^{-6}, replacement would require 5000 gigawatts. That amount of power would easily be generated by an array of solar cells forming a square about 150 km on a side.

A 10^{14}GeV electron linac would need to be about half a lightyear long (assuming 10^8 volts every 4 meters, as is achieved by recent 10km-long designs). Even a 10^{19}GeV linac would only need to be 50,000 light years long. (Although this is gigantic, it is only about 1/1000 of the diameter of our galaxy, and, even assuming a mass of 10 tons per meter of linac length, the total mass of the 50,000 lightyear linac would still be slightly smaller than the mass of our planet. Similarly, the power requirements of this linac would be well below the power output of our sun.) The vacuum inside the beam tube would have to be of order 10^{-23} atmospheres in the former case and 10^{-28} in the latter, which is comparable to the vacuum in intergalactic space.

3.4 Gacs' reliable 1D cellular automaton

Gacs [12] considered the question of how to convert any 1D cellular automaton to a functionally equivalent "reliable" 1D cellular automaton.

Here is an extremely crude sketch of Gacs' results and methods. A "1D probabilistic cellular automaton" (PCA) is a 1D array of N identical cells. Generally one considers a bidirectionally infinite array ($N = \infty$), but Gacs also allows finite N with periodic boundary conditions. Each cell can be in any "state" selected from a fixed finite set $\mathbf{S}$. Each timestep, cell x transitions from its current state $s_{x,t} \in \mathbf{S}$ to its new state $s_{x,t+1} \in \mathbf{S}$ according to a function (the "transition function") of: $s_{x-\ell,t}$, $s_{x+1-\ell,t}$,..., $s_{x-1,t}$, $s_{x,t}$, $s_{x+1,t}$,..., $s_{x+\ell-1,t}$, $s_{x+\ell,t}$ (where ℓ is a constant) *and* of a random real number $r_{x,t}$ uniform in $[0, 1]$. (The random numbers are generated anew, independently, each timestep and for each cell.) A "1D deterministic cellular automaton" (CA) is the same except the transition function does not depend on any random r. A CA "simulates" another if each possible cell-state in the simulated CA corresponds to a subset of the cell-states permitted in the simulator CA (and the subsets corresponding to different states, are disjoint); there must also be an easy-to-follow recipe for generating the initial ($t = 0$) state of the simulator CA from a known initial state for the simulated CA. It is well known [21] that a CA can simulate a UTM. A PCA is "noisy" if every possible output cell state arises with a nonzero probability in its transition function. A PCA is "within ϵ" of another with the same cell-state set if all the probabilities of cell output states in the first CA's transition function (given the same predecessor states) are within ϵ of the probabilities of the corresponding output states in the second's.

Gacs' main result is this. Suppose we are given any particular 1D CA C_1. Then there exists a second CA C_2 which simulates C_1 and such that, for any sufficiently small $\epsilon > 0$ (the value of the supremal permissible ϵ may depend on C_1, but for any particular C_1 is a positive constant) any noisy PCA C_3 within ϵ of C_2 will have this property: For any particular x and $t > 0$: the probability measure of the "bad$_x$" subset of the spatial states of C_3 at time t (i.e. of those that disagree at cell x with the spatial state of C_2) is $< K\epsilon$ for some constant K (perhaps depending on C_1, but not on x or t).

Gacs' result also may be formulated for finitely-long CA's with N cells, and is essentially the same except that the upper bound $K\epsilon$ is replaced by $K\epsilon + t\epsilon^H$ where $H = N^{O(1/\log\log N)}$. I.e., reliability lasts for time growing almost exponentially with N.

Gacs also formulated and proved corresponding results for continuous-time analogues of CAs, in which instead of a transition function we have a "transition-rate function" and each cell is always deciding randomly whether to make a transition at any moment in continuous time (with these decisions being independent both of its previous decisions and of the decisions made by other cells).

The method of proof involves an infinite hierarchy of simulators. In the first level the hierarchy, local "colonies" of Q cells store redundant information concerning a single cell in the lowest-level CA, and attempt to enhance that cell's reliability via redundancy and error correction coding. At the kth level, Q^k cells store redundant information about Q^{k-1} cells. This all must be accomplished, for all $k = 1, 2, \ldots, \infty$ simultaneously, with only a constant factor in overall space-redundancy and time-overhead, and in such a way the the reliabilities converge to 100%, and in such a way that the hierarchy "self-organizes" from an initially homogeneous (except for a finite amount of "input data") state.

4 Problems our design still suffers from

Our Universal Cellular Automaton must be initialized in a precise logical state (a certain infinite periodic pattern). Indeed our whole universe must magically be created, and it must satisfy some rather special boundary conditions at spatial infinity. These are just the "rules of the game" and could not be expected to be avoided.

Presumably our computer would, due to thermal diffusion (or quantum tunnelling) of atoms altering the shapes of its parts (if for no other reason), eventually break. It could continually repair and rebuild itself by means of robotic factories. However, we admit that this has not really been proven possible – since Gacs' error correction theorem lives in a rather artificial simplified model[13] which may not capture the true complexities of robotic self repair. For example, a deranged part of our CA could get the idea that it should use its factories to build robot warriors, whose mission would be to conquer neighboring CA territory, destroy it, and rebuild it converted to the mission of Jihad. Though such a dementia would be exceedingly unlikely, in an infinite CA, it would be *certain* to happen in an infinite number of places. Plausibly, in our larger model, some such Jihads *must* be successful, since every possible kind of warrior subculture will occur someplace, and unless our intended computation includes, as part of its program, commands to build the maximally-tough kind of warriors, the "good guys" eventually will be defeated, or at least battled to a draw (since the identical warriors will occur someplace else, only devoted to the cause of "evil"). One could counterargue that such warrior subcultures would arise exceedingly rarely and would be surrounded by enormous expanses of "good guys" – suggesting they could always be defeated. The answer is not clear, so in this sense our demonstration is incomplete.

Gacs' theorem shows that *in his restricted model*, such Jihads could not be successful, but the problem is that, physically, the true cell-state space cannot be restricted to only the states Gacs wants – there also exist all sorts of physically possible CA cell-states which Gacs' CA designs would never have permitted. The problem is that, physically, one quite likely cannot build even a single cell of almost any CA, because any real design would actually have a much larger number of physically-accessible states that the asked-for number, and the transition structure within those extra states would be essentially uncontrollable. I.e., Gacs' noise model is actually not a realistic model of noise: genuine noise is not so polite as to only transition us to a designed small finite set of states whose transition structure is also designed. Real noise instead takes you a very *large* set of states (all physically accessible ones) whose transition structure is set by the laws of physics and hence cannot *be* designed! This might be viewed as a fundamental shortcoming of Gacs' whole model which should prevent its applicability in many scenarios. This objection has not been pointed out previously. So really, Gacs' reliability enhancing methods should be viewed as noise-immune *only* if the noise only *rarely* causes a state change *and* if the state changes it does cause are never "severe" changes that move outside of the designed state set. Gacs calls these "hardware errors" and forbids them.

It is impossible to extend Gacs' theorem to allow all kinds of hardware errors. To see that, realize that arbitrarily long blocks of cells with hardware errors will certainly ultimately arise, and those cells would serve to bisect our CA. If the undesigned transition function is such that broken cells can stay broken if all of their left-neighbors (or all of their right-neighbors) are broken, then there would be nothing that could be done. It would be interesting to understand just what kinds of hardware errors could be permitted.

Another possible problem is instability. It appears that our rods are stable to all small axially symmetric perturbations which "dig a hole" in the surface of our rod-shaped "planet" and "pile the dirt in a hill." That is because it is energetically favored for the hill to fall back into the hole. Similarly, perturbations which, e.g., change the cross-section of the rod from an exact circle to a near-circular ellipse, should be energetically disfavored.

The kind of small perturbation that worries me is *bending* of the rod. If we temporarily ignore General Relativity, then the well known analysis of "buckling of columns" first done by Leonhard Euler shows that any sufficiently long fixed-radius column, made of any material with finite elastic moduli, and subjected to *any* fixed nonzero compressive load, is unstable to sine-wave buckling at any sufficiently long wavelength.

That makes it plausible that general relativistic rod solutions are also unstable (in the same, or perhaps in other, ways). But considering how severely cylindrically symmetric GR solutions have already violated our Newtonian intuition, this must be regarded as an open question[14]. Even if there is buckling instability, it perhaps would be possible for our UTM to defend itself by, e.g. emitting more heat in one direction than the other to move any desired part of the rod in any desired direction. Again, it is an open question whether such "defense" is possible forever.

A related problem is this. Suppose a conventional (Schwarzschild, roughly spherical) black hole is created somewhere along our rod, either by deranged warriors or by some exceedingly unlikely huge thermal fluctuation. Could this black hole then eat its way along our rod forever, destroying everything? The dynamics of what would happen are again a completely open question[15].

Throughout the whole argument of this paper, we have taken for granted numerous "well known engineering facts" without, however, proving them from axioms about physics. Thus our "proof" cannot be regarded as a completed theorem of mathematics. Probably such a proof is beyond reach in the forseeable future. Still, though, it is clear that we have overcome what appeared to be the major physical obstacles to such a proof, and we believe we understand the ways in which we have fallen short. My personal opinion (in view of the objections above) is that we *have* fallen short and indeed that a Turing machine cannot exist with the present laws of physics. (It is not even clear how one could formulate such a "nonexistence theorem," though, much less prove one.) However, with slightly "nicer" laws of physics, e.g. in which hardware decay could not occur (only "software" errors could) because

[13]Nevertheless, Gacs' model has very great generality.

[14]I am unaware of any stability analysis of any nonflat cylindrical GR solution, or any GR solution involving inhomogenous matter, whatever. One could also consider the effect of allowing a nonzero cosmical constant on all this.

[15]This is not a question about stability with respect to infinitesimal fluctuations, as in Euler's column-buckling analysis. This is a question about a very *large* fluctuation.

quantum and thermal effects were somehow kept bounded[16] well below damage threshholds, then it appears that a Turing machine could exist. I.e., if the laws of physics were general relativity with *classical* never-decaying *continuum* matter and *bounded* noise, then, I claim that (at least if we ignore issues related to possible buckling instabilities) we have genuinely proven UTM existence (or that, at least, the remaining details of such a theorem and proof could easily be filled in). Also, it appears an infinite-rod "planet" supporting an infinite, infinitely long-lived civilization similarly could exist – *provided* the inhabitants of that civilization agreed to behave responsibly! We have provided them with an infinite place to live, a permanent power source, and permanent cooling, and shown them how to compute anything computable, even in the presence of noise, provided that noise never is too severe anywhere. The rest is up to them! One might conjecture that Turing would have felt at least some partial satisfaction if he had known about all this.

5 Acknowledgment, and remark on dissipationless reversible computers

Thanks to Michael P. Frank for some helpful comments. Frank originally became interested in the problem because of his interest in *reversible* computation. Let me make some remarks about that. Our arguments in §2 suggest that a UTM must be 1D, e.g. rod shaped. Any such computer at constant temperature (which the Bennett [1] "thermal" reversible computers require) must receive power input large enough to compensate for the thermal radiation (constant power per unit length) it gives off. Hence from the standpoint of power and heat, using this kind of reversible "dissipationless" computer actually would not make our design task any easier than by using conventional irreversible computers![17]

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[16] By decoherence effects, and extreme cold combined with nonlinear effects that cut off the high-energy tails of thermal distributions, respectively? Or because of the availability of new magic construction materials made of genuine continuum-matter instead of atoms?

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