Invisibility is Uncomfortable

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Abstract

As a consequence of information-theoretic limitations arising from thermodynamics and the laws of physics, derived in a previous paper, we conclude that: the sustained wearing of a "cloak of invisibility" is impossible. Although this is a fundamental "unsolvable" computational problem for computers which obey thermodynamics, analogous to the "halting problem" for Turing machines, I doubt that the present problem is as central here as the halting problem is there.

KEYWORDS: thermodynamics, computation, cloak of invisibility, blackbody radiation, entropy.

1 The problem

Imagine two concentric spheres, both surrounding your body. The region between the two spheres is supposed to contain a "cloak of invisibility." While wearing the cloak, you, and the cloak, are supposed to be invisible to any outside observer. That is, this outside observer sees only whatever he would have seen, if you and the cloak had not been there.

The outside observer, in his attempt to see you, is allowed to look at you from various angles and to set up various light sources that he can later stare at through you.

Is such a cloak possible, even in principle? The answer is "no." (Provided that certain restrictions are imposed and certain ground rules are obeyed by both parties. We will soon discuss these.) This is an amusing specific consequence of certain fundamental limits on computation, arising from thermodynamics and laws of physics, which had been derived in a previous paper by the same author (*Fundamental physical limits on computation*, NECI TM June 1992). The essential point we use is that blackbody-like radiation contains the most possible information flux at a given power-flux, and attempts to transmit more information with less power (e.g. by "recycling" the power) cannot work[1].

2 Rules of the game

The location of the observer (or observers) is not known to the wearer of the cloak. For example, the "observer" is allowed to set up a large sphere enclosing the cloak, whose surface is completely coated with photon emitters and detectors. In this case the question of the "location" of the observer would become silly.

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[1] We avoid issues related to transmission through media here. More information could actually be transmitted using less power in an optically dense medium, or via sound waves in a solid material, than by photons in vacuum, but (we argued in the previous paper) such schemes will not work at sufficiently high fluxes because one runs out of phonon modes. We are going to assume that the information fluxes we are dealing with are neither too low or too high, so that the result from the previous paper holds. As was argued there, this regime of validity roughly corresponds to power fluxes in the 24-order power-flux range corresponding to a blackbody radiation from a temperature-T body, 10^3 Kelvin $< T < 10^9$ Kelvin. Also, as was argued in the previous paper, more instantaneous information flux is possible by storing up information and releasing it in a short burst, but over the long term, such tricks cannot work. Actually even if we were *not* in this regime of validity, the arguments of this paper would *still* work to rule out cloaks if the ground rules were changed to allow the observer to *also* use phonons, media, etc. The H/P formula would no longer be valid, though.

The wearer of the cloak has an external source of arbitrarily large amounts of power.

The observer is allowed to generate and remember "random" bits (i.e. bits which could not be known to the cloak wearer).

The observer is not allowed to use any means other than photons (e.g. neutrinos, gravity) at bearable intensities. For example he is not allowed to explode a large bomb near the cloak, destroying the cloak and therefore making its remnants become visible! Just what "bearable" intensities are is a little vague, but suffice it to say that we will not allow arbitrarily high power. And we will assume that the cloak and its occupant actually *are* arbitrarily close to perfectly transparent to all photons outside of a certain "reasonable" wavelength range.

H.G. Wells in his early science fiction novel "The invisible man" proposed that people could become invisible by simply (1) removing all pigments from their body by use of bleaches (2) taking drugs which would cause their index of refraction to become the same as that of the surrounding air, and (3) removing their clothes. We will assume that any such technique would kill any human being and that this would be unacceptable. In short, the cloak is worn by a being who has quite non-invisible optical properties and finite size, and who refuses to alter either these properties or his size. The cloak is allowed to be only a constant factor larger (at most) than the size of its occupant. The cloak therefore must work by routing incoming light (or information equivalent to that light) around the cloak's wearer.

Finally, since light (or information) cannot take a straight line path through the cloak's occupant, but must be routed around him or her, the time of flight of the information will of course be longer than it should have been, which leads to an easy way to detect any cloak. This is too boring a detection method, so we will specify as a ground rule that the observer is not allowed to measure time of flight, nor is he allowed (what would allow the same trick) to transmit two photons along two separate paths, one of which is through the cloak, one of which is not.

3 The Argument

Send information, as uniform blackbody-bath-like radiation, toward one hemisphere of the cloak and measure what is emitted from the other. The two should match, i.e. the spacetime history of the radiation field should correspond to the "same" point in high-dimensional "phase space," up to unavoidable quantum uncertainty. If they don't match, then you immediately know somebody is there. So assume they do match because the cloak works.

If the cloak and its wearer are present, then in some region inside the cloak, information must be flowing at a flux at least some constant factor (>1) larger than the flux in the blackbody radiation itself (since it is being routed around the wearer). It is impossible for there sustainably to be such a region, unless the cloak is irreversibly using up power – and more power than is available from the incoming radiation – which is converted to heat. This result on information flux versus power usage comes from the previous paper. The precise conditions under which this is valid are discussed at length there and won't be discussed here. In that case, the cloak will eventually have to radiate this heat power, at which point it will become detectable as an object at distinctly higher (a constant factor higher) temperature than the pseudo-temperature of the blackbody-like radiation it is being bathed in.

The argument is now complete. Note that a cloak perhaps *is* possible if one only is required to wear it for some finite period of time and then gets to dump heat at the end of this time. The present argument only rules out the possibility of a cloak that has to keep working for an arbitrarily long period of time. This explains the title of this paper, for which I thank Henry Cejtin.

Also, if the cloak's wearer had a way to invisibly dump heat at some rate H, and the observer can afford to irradiate the cloak with power P, then the present argument will rule out the possibility that the cloak will work forever, if

$$\frac{H}{P} < (\frac{A}{A-a})^{1/3} - 1$$

where A is the surface area of the cloak and a the surface area of the occupant, and both are spherical. But if H/P were larger than this, a cloak is (perhaps) possible.

Craig Helfgott pointed out to me (and his suggestion resembles an idea from the science fiction novel *Stardiver* by David Brin) that, if the observer were incapable of detecting, some wavelength range, e.g. X-rays, then it would be possible for the cloak wearer to "dump the heat" (which would otherwise necessarily make him visible) in the form of fluctuations in an X-ray beam. I agree: assuming a future ridiculously advanced X-ray refrigeration technology and a big enough power supply, this could indeed work, or anyhow could not be forbidden on purely thermodynamic grounds. So we indeed need to assume that the observer can detect all wavelengths going in all direction, or at least, all wavelengths the cloak wearer is capable of emitting for possible cooling purposes.

4 A promising argument we didn't use

Another scheme which might be very effective for unmasking invisible people is similar to, but simpler than, C. Bennett et al.'s "quantum cryptography." (See for example *Experimental quantum cryptography*, J. Cryptology, 5, 3 (1992).) The observer transmits a sequence of photons where photon number i is polarized at angle θ_i. He then receives the photons through a polarizer at angle θ_i. The observer knows the θ_i's but the cloak wearer cannot. One cannot measure the "angle" of polarization of a photon, one can merely see if it is spin $+1$, -1, or 0 in some chosen direction. Hence, the cloak wearer, if he is measuring incoming photons and retransmitting simulated ones, will eventually have to do something wrong and the observer will fail to receive a photon eventually.

So the cloak wearer must never be measuring the photons... whatever "measuring" means. This issue (meaning of measurement) is the crux. It is not clear to me that receiving and retransmitting a photon, *without* ever remembering anything about it, is a forbidden "measurement." So it is not clear to me that this argument works. If it did work, though, it would rule out the existence of invisibility cloaks, no matter how much power and cooling were available, and the unmasking would be very fast.