Ants, Bees, and Computers agree Range Voting is best single-winner system

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***Abstract* — In *Range Voting* (RV), voters give each candidate a score from (say) 0 to 9; highest average wins. (Voters may leave some scores intentionally blank, which are not incorporated into the averaging; then a "quorum rule" is commonly added to prevent electing candidates with too few genuine scores.) Two lines of evidence indicate RV is the best voting system among common simple proposals:**

1. Computer measurements of "Bayesian regret" (BR) of many voting systems in many probabilistic models, find RV robustly has the least BR. In both these and retrospective examinations of historical national presidential elections, the quantitative advantage RV exhibits over "plurality voting" is comparable to plurality's over "random winner."
2. Honeybees (*Apis Mellifera*) have made $\approx 10^{16}$ democratic hive relocation decisions. "Reverse engineering" videotaped bee-elections shows Darwinian evolution found a method essentially equivalent to RV. *T.Albipennis* ants also apparently use RV.

Keywords — Democracy, single winner voting systems, Bayesian regret, computer simulation, bees, ants, parallel algorithms.

1 Introduction

Arrow's Nobel-winning 1951 "impossibility theorem" [9] misdirected the entire field of voting systems for 50 years. It showed no single-winner voting system could exist satisfying a certain set of apparently-desirable properties – commonly interpreted as showing no "best" voting system could exist.[1] But:

(a) Arrow's definition of "voting system" was very restrictive. It excludes range voting, and an artificial related system called "honest utility voting" *achieves* Arrow's "impossibility" by simultaneously satisfying all his "incompatible" criteria.

(b) It is subjective and eternally debatable how "desirable" Arrow's (or anyone's) properties actually are.

(c) (most important): The entire idea of evaluating voting systems by simply listing the logical "properties" they always satisfy, while educational, is misguided. Rare "paradoxes" are irrelevant in practice – especially "paradoxes" only subjectively bothering *some* people. We instead want an *objective quantitative* measurement of *how much* that voting system's flaws hurt society by electing suboptimal winners, *averaged* over *all* election situations from a realistic probability distribution.

There are three ways to make such a quantitative evaluation (the first being less desirable):

1. Examine actual elections throughout history (or artificial ones conducted by paid experimental subjects), try to determine what results would have happened under different voting systems and how "good" each would have been. For genuine elections, this is extremely subjective and debatable. With artificial elections with paid subjects, this is expensive, arguably unrealistic, and so far zero such studies have ever employed range voting and hence (not coincidentally) have failed to find any "best" system. Either way, we get an extremely small dataset so that few statistically significant conclusions can be drawn. Even the strength of this approach – the fact it is based on the right sample-space (genuine human elections) – can be turned against it since quite possibly that sample-space would alter if different voting methods were used.[2]

2. Conduct elections via computer simulation. The computer generates artificial voters and candidates, and Bayesian utilities indicating how "good" electing each candidate would be for each voter. It then conducts elections (under certain assumptions about the behavior, utility-knowledge, and electorate-knowledge of voters, and using whatever voting systems one cares to program), finds the election winners, and retrospectively evaluates the utility of those election winners summed over those voters. Advantages: cheap; easy to get an enormous amount of data and hence conclusions with negligible statistical error. Although real human voters mislead pollsters, *simulated* voters (whose "minds" can be "read") do not. This can be attacked on the grounds that the models simulated may be "unrealistic." But many can be tried, getting coverage of a wide space of possibilities, at least some of which are likely to be realistic – so that if (as will occur here) *all* the models agree on some conclusion, that is very strong evidence for it.

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[1] Because: By Arrow's theorem, all voting systems inherently exhibit strange "paradoxes." So in an election-situation designed to stimulate voting system X's flaw, X will perform less well than some other system Y, and since this is true for all X, it is true for any putative "best" one, thus proving there is no best system, Q.E.D.

[2] "Duverger's law" indicates that some voting systems (plurality, instant runoff) lead to self-reinforcing 2-party domination, while others (plurality with separate top-2 runoff round, and probably also range voting) lead to more than two stable parties.

3. Create some eusocial lifeform which, as an important part of its lifecycle, holds a single-winner election, and such that the choice made by that election has a major effect on Darwinian fitness. Let it evolve on some planet for 100 Myr while conducting over 10^{15} elections – then see what voting procedure was invented and how well it performs! This has the advantage of complete realism and unbiasedness – no human notions of "best" or "moral fairness" input whatever. This experiment has *already been done* by honeybees and ants on Earth. This could be attacked on the grounds that perhaps insects do not have the brainpower to use some of the more-complex human-invented voting methods and hence that this experiment *really* can only hope to identify the best method attainable with *bee-brainpower*. We respond that most humans won't *accept* voting methods too complicated for bees,[3] in which case for most practical political purposes this attack is irrelevant.

Here are some rough estimates about the **number of elections**[4] of each kind:

1. Formal human elections throughout history: 10^7;
2. Computer election studies conductible in 1 CPU year (GHz machine): 10^8;
3. Bee elections[5] so far: 10^{16}.

Range voting (RV) is described in the Abstract. E.g, a valid range vote in the 2000 US presidential election would have been "Browne=3, Bush=0, Gore=9, Nader=9, Phillips=X" where X denotes intentional blank. We shall see that, of our three approaches above for quantitative comparison of different voting systems, the last two both support the notion RV is the best system – and the first appears also to support it, although much less clearly.

All three approaches are subject to this **criticism:** Since there are an infinity of possible voting systems but only finite computing time, we cannot guarantee finding the truly "best" voting system – we can merely hope to find the best *among the finite set of alternatives* which humans, computers, or Darwinian evolutionary experimentation happen to have invented.

A **4**th approach – proving mathematical theorems – conceivably could avoid that criticism, but so far that's only been possible in idealized unrealistic mathematical "worlds." One noteworthy such invention (by economists, not political scientists!) was "Clarke-Tideman-Tullock" (**CTT**) voting [31][34][35] in which voters *pay money.* CTT can be *proven* "perfect" in the sense that idealized isolated rational economic animals will always vote "honestly," and if so (and assuming voters have perfect knowledge about utilities) the utility-sum-maximizing alternative for society automatically is elected. Unfortunately, since voting in modern democracies may be the foremost example of the *huge* falsity of modeling humans as rational economic animals [5], and since humans are not isolated but instead can communicate and collude, real-world "perfection" of CTT voting cannot be expected. (Also, "voting systems" with money payments are unconstitutional in the USA.)

2 Actual human elections

The largest dataset of apparently "wrong way" (due to mathematical flaws in the voting system) national presidential elections seems to be mine [31].

Despite the large subjectiveness of any such exercise, we still claim that examining that collection makes it clear the plain "plurality" voting system[6] elects "wrong winners"[7] in one election in ≈ 8. The two-round plurality *plus* (if nobody got over 50%) top-two "runoff" second round system, appears to perform better – very roughly, a 5-times smaller rate of clearly identifiable "wrong winners" – at the cost that *two* elections often must be held. But multi-round systems still clearly have delivered "wrong winners" in two cases (Chile 1970 & Peru 2006) plus unclearly perhaps in several more. Highly probably the "approval voting" system[8] would have delivered a wrong winner in Portugal 1986 (as an instance of J.H.Nagel's "Burr's dilemma" [22]) so it too is no cure-all.

It is *plausible* (but certainly not "provable") from this data that RV would have done much better. I know of no historical national-presidential-level example where RV would have delivered a clear "wrong winner."

We can now make a **crude quantitative estimate** of how much societies would benefit from switching to better voting systems. If *plurality* voting leads to one wrong winner per 8 elections, plus the voters themselves select two further wrong choices (e.g. US President Warren Harding, elected in 1920 by a 60.3% majority "landslide" but the most common choice among historians for "worst in US history"), that represents ≈ 5 "forward" and 3 "backward" steps for net forward progress *2 steps.* A *superior* voting system might yield 6 forward and 2 backward steps for net forward progress *4 steps.* Finally, with random coin flips (representing nondemocratic choice) choosing the winner from (as is common in US politics) *two* choices, we expect forward and backward progress to exactly cancel to *zero.* So,

[3] Our polling [32] revealed the majority of USA voters prefer plurality to RV, generally on the grounds that RV is "too complicated," despite RV's greater simplicity versus most proposed systems.

[4] We only count elections with at least 2500 voters.

[5] Eusocial stingless bees have existed for at least 65 (and [11] honeybees for at least 22-25) Myr [7][19] but there is disputed ([8] p.151) evidence [12][13] that bees may have existed 220 Myr ago, *before* flowers. The highly eusocial bees, living in large hives with tens of thousands of inhabitants for multiyear tenancies with the vast majority of all reproduction being done by a single queen, include about 10 honeybee and about 200 stingless bee species. Honeybees are more studied. Eusocial bees are found in every continent save Antarctica. In 1996 there were 300K registered beehives in New Zealand (area 269 K sq km) extrapolating to 166M beehives worldwide. At 1 election per beehive per year for 40 Myr, we get 7×10^{15} bee-elections, which rounds up slightly (since unregistered hives exist!) to 10^{16}. So there have been between 5×10^{14} and 10^{17} bee-elections.

[6] Plurality: name one candidate as your vote – most named candidate wins.

[7] Defined intentionally vaguely as "not what the voters most wanted."

[8] AV: name the subset of candidates you "approve" as your vote; most-approved candidate wins [3].

As a crude estimate, we expect the benefit from switching from plurality to a good voting system, to be comparable to the benefit got by the original switch from nondemocratic government represented by "random winner" to plurality based democracy.

3 Computer simulations measuring Bayesian Regrets

Suppose each of V voters has some "utility" $U \in \mathbb{R}$ for the election of each of C candidates (VC in all). In computer simulations, the Us would have to be pre-generated by some randomized algorithm "utility generator." Let the voters then cast votes which cause election system E to select some winner W. Each vote could either arise "honestly" from the C utilities; or instead from some "strategy" also based on knowledge about "pre-election polls" of a random subsample of a superset of the electorate; and in either case the voters could act based on Us *distorted* by "noise" or "ignorance." This winner W might be the "best" candidate – maximizing utility averaged over all V voters – or might be somebody else, in which case that election exhibits "regret," which is, quantitatively, the average U difference between the best and actual winners.

Define the **Bayesian regret** (BR) of voting system E to be the expected regret exhibited by E. Smaller regret is better – the hypothetical "magic selection of best winner" system achieves BR=0. Another useful artificial benchmark voting system is "random winner."

BR is a function not only of E, but also of the numbers V of voters and C of candidates, the utility generator ("issue-based"? Just independent random numbers?), and the assumptions about voter behavior (honest, strategic, ignorant, etc.), Once these "knob settings" have been chosen, BR may be evaluated to arbitrary accuracy for any proposed single-winner voting method E via Monte Carlo experiments.

My 1999-2000 computer experiments [30] remain the world's largest – in terms of the number not only of elections (each BR datapoint was based on between 10^5 and 2×10^7 elections), but also of voting systems, utility generators, and voter-behavior assumptions tried. Nevertheless, this study has been criticized as not being extensive enough in all these dimensions, hence an even-larger study would be desirable. Mine also was the first such study to include range voting as a contender, and hence (not coincidentally) the first to reach any clear conclusion that any of the voting systems compared, was "best."

A full printout of the BR values output by the program would require a book hundreds of pages long. However, they can be **summarized in only one sentence:** *In every one of 720 "knob setting" probabilistic scenarios, range voting came out best (or co-equal best) by the BR yardstick, ignoring differences too small to be statistically significant.*

Although any one of the 720 could be knocked as "politically unrealistic," the net force of RV's superiority in *all* of them is imposing. One might even proclaim it "one of the most strongly supported experimental conclusions in all of political science." We now describe most of the voting systems, voter-behavior assumptions, utility generators, and other knob-settings:

Voting systems [23][21][9,10] and voter-behaviors:

0. Honest range voting (utilities as scores, but linearly transformed so best-utility candidate gets 100, worst gets 0, and rest linearly interpolated).
1. Honest Borda voting (voters rank candidates 0 to $C-1$ best-to-worst; candidate with least sum-of-rankings wins).
2. Honest Condorcet Least-Reversal (LR) voting (voters rank candidates best-to-worst; if a "beats all" candidate preferred pairwise over each opponent exists, he wins; otherwise elect the one such that the smallest number of pairwise-preference relations would need to reverse, to convert him to a beats-all winner).
3. Honest Coombs STV (the candidate bottom-ranked by the most voters is eliminated each round until only one – the winner – remains).
4. Honest IRV ("instant runoff" voting; the candidate top-ranked by the fewest voters is eliminated each round).
5. Honest Copeland (the winner of most pairwise elections wins).
6. Honest Dabagh [6] point-and-a-half voting (voters give one point to their favorite and half a point to their second-favorite; candidate with greatest sum wins).
7. Honest Black [1] (if no Condorcet "beats-all" winner, use Borda).
8. Honest Bucklin (voters rank candidates; initially $k = 1$; examine only the top-k choices made by each voter, counting each name as having a "vote" [kV votes total]; keep incrementing k until the most-voted-for candidate has $> V/2$ votes and then he wins).
9. Honest plurality + runoff for 2 top finishers (voters name a single candidate as their vote; the top-2 then compared in a *second* election and the one with the most votes in this runoff, wins).
10. Honest plurality (1 vote for max-utility candidate)
11. Honest antiplurality (1 vote against min-utility candidate)
12. Majority vote on random candidate pair, all $C-2$ other candidates ignored. (A system invented by A.Gibbard.)
13. Random dictator (chosen from voters) dictates winner. (Also invented by Gibbard.)
14. Random winner (artificial system≈non-democracy).
15. Worst-summed-utility winner (artificial system).
16. Honest approval voting (using approval-threshold = average-candidate-utility; the most approved candidate wins).
17. Strategic range/approval voting (average of 2 frontrunner utilities is threshold; approved candidates receive maximum and disapproved candidates minimum possible range vote)
18. "Rational" range/approval (threshold for $k+1$st candidate is moving average of the first k candidates, considered in most-to-least-popular order according to pre-election polls)
19. Rational plurality (vote for the least-worst of two pre-election poll frontrunners).
20. Strategic Borda I (rank one poll-frontrunner top, other bottom, rest recursively).
21. Rational antiplurality voting (vote against the worst of the two poll-frontrunners).
22. Strategic Condorcet LR (voters' strategy same as in 26).
23. Strategic IRV (strategy in 26).
24. Rational Borda (1 frontrunner ranked top, other bottom, rest using moving average to decide if top or bottom within still-available ranking slots)
25. Strategic Coombs STV (strategy same as 26).
26. Strategic Borda II (rank one frontrunner top, other bottom, rest honest)
27. Rational Dabagh point-and-a-half (moving average as threshold, give top still-available score to candidates above the threshold).
28. Strategic Copeland (strat. 26).
29. Strategic Black (strat. 26).

[9] Note our artificial treatment of different voter behaviors under the same voting system as "different voting systems." This, although permissible, was a software-design mistake that caused the programming labor to do K voting systems with B different behavior models to grow proportionally to KB. Future larger and better studies should redesign to make programming labor grow only proportionally to $K+B$.

[10] Trying several more systems, e.g. "Nauru," "Cretney," "Tideman ranked pairs," didn't alter our conclusions.

Number C of candidates & V of voters: $C \in \{2, 3, 4, 5\}$ and $V \in \{5, 10, 20, 50, 100, 200\}$ for 4×6 different choices in all. Based on preliminary experiments and extrapolations, values $V > 200$ did not seem to change the nature of our results, hence were excluded as consuming too much computer time.[11] Note that when $C = 2$, achievable voting systems will *not* achieve zero BR. That error made in a previous study [17] probably indicates it had a computer programming "bug."[12]

Utility generator: Each candidate and voter were assigned random-uniform values in $[-1, 1]$ as their "stances" on each of I issues. The utility of a candidate to a voter was the inner product of their two I-vectors, plus a random-uniform deviate in $[-1, 1]$. All utilities were then divided by $2I + 2$ to normalize them to lie in a unit-width interval. We used 6 different utility generators: $I \in \{0, 1, 2, 3, 4, \infty\}$, where "$I \to \infty$" means table 4.1's "$VC$ independent random normal utilities" model.

Voter ignorance/delusion levels: With "ignorance," each voter does *not know* his own candidate-election utilities but instead a version *polluted* by added noise. Specifically we add Gaussian random deviates with mean 0 and standard deviation Q to each mental utility value.[13] (If $Q = 0$ this reduces to the old, ignorance-free, program version.)

As we've just described it, voters have *identical* probability distributions of their ignorance for all candidates. But in practice, understanding of candidates is candidate-dependent. To model that, we further modified the simulator to make Q_j depend on the candidate j, i.e. itself be a random variable uniform on $[0, Q]$ chosen once per election. The ignorance levels tried were $Q \in \{0, 0.49, 0.99\}$ and the latter two Q-values were tried with both candidate-independent and -dependent ignorance, for 5 different ignorance-possibilities in all.

Summary. We tried $4 \times 6 \times 6 \times 5 = 720$ knob-setting combinations in all. For each, 30 different voting systems were tried, yielding $30 \times 720 = 21600$ different BR values, each based on $\geq 10^5$ elections.

All previous BR studies [17][2] found that some voting systems were better in some situations but worse in others, i.e. failed to reach any clear conclusion one system was "best." (E.g, Borda with honest voters is better than Approval voting when there are sufficiently many candidates, but Approval is superior with strategic voters or few candidates.) No previous study had included either ignorant or strategic voters, used documented random number generators, or made the source code for their program available to the public. Ours was the *first to include range voting as a contender*, hence was the one to discover RV's robust superiority, both for honest and strategic voters, for numerous election sizes, and with numerous "ignorance levels" and utility generators. But RV's superiority is less pronounced the greater the voter ignorance level.[14]

Many years afterwards, I programmed and ran two much smaller and less ambitious (but simpler) supplementary computer simulation studies, one an exhaustion of the "left-middle-right 3-candidate 1-dimensional politics scenario" [33] and one for "honest-voter random normal elections" (see table 4.1 and [31]). These also concluded RV was robustly best. The former supplementary study was the first to try a 50-50 *mixture* of "honest" and "strategic" range voters (which an exit-poll study [32] found was a decent approximation to what actual human range voters do), and found, amazingly fortuitously, that that actually yielded *lower* Bayesian Regret values than with either pure-honest or pure-strategic voters. This contradicted the usual experience that the more honest the voters, the better voting systems perform (as measured by BR) – with the single exception that at many knob settings, dishonest voter-strategy actually improves *plurality* voting's BR versus all-honest voters. Colorado electrical engineer Peter L. Bielawski tells me he too conducted his own computer simulations, also concluding RV was the best system he tried (but Bielawski did not provide any further details).

Because BR is *quantitative*, we can enquire by *how much* RV improves over other single-winner systems. The regret reduction got by switching from plurality to RV usually is a *factor* of 7-10 for honest voters and 2-3 for strategic ones. This *exceeds* the improvement-factor got by switching from non-democratic systems such as monarchy (assuming monarchs, on average, were as least as good rulers as random candidates, since, e.g. they were trained from birth to rule), to plurality: only a factor of 2-4 for honest voters and ≈ 1.6 for strategic ones.

For those who prefer additive differences over multiplicative factors: the improvement we got from switching from random winner to (the USA's current) strategic plurality system is comparable to what we should get from a future switch to RV with an approximate 50-50 mix of honest and strategic voters – *assuming* this mix yields Bayesian regrets midway between the pure-honest and pure-strategic values; if the "amazingly fortuitous" regret-bonus seen in the left-middle-right study [33] instead happens, there would be a larger win.

To conclude:[15] based on Bayesian regret measurements, *switching from plurality-based democracy to range-voting-based democracy should yield a comparable or larger improvement than the original* <u>*invention*</u> *of democracy.*

4 Honeybees

Honeybees have performed vastly more elections than either humans or what computer simulations can equal with present technology for reasonable money. And those elections were about an issue far more important to each bee than human

[11] "Pre-election polls" were done using a *separate* pool of "voters" generated in the same way, i.e. were a *co*sample not a *sub*sample.

[12] Note that honest range voting also does *not* achieve BR=0, contrary to assertions made by too-quick readers of my original [30].

[13] Allowing non-zero means would have made no difference if these means were candidate-independent. If they were candidate-dependent (but i.i.d.) random variables, then that would merely have had the effect of making the statistical noise worse.

[14] In the limit of enormous ignorance, all systems become equivalent to random-winner.

[15] And confirming our extremely crude estimate from §2.

elections are to most humans – the location of the bees' new hive – and conducted in real-world conditions. So it behooves us Johny-come-lately democrats to solicit advice from bees.

Each spring, about half the inhabitants of each honeybee hive leave with their queen to start a new hive, in a $\approx$ 10-Kbee swarm. The most important decision they need to make is: where to build that new hive? They usually find about 20 different options within $\approx$ 100 sq km, and about 90% of the time, the bee swarm succeeds in selecting (what entomologists think is) the best. If so, this performance seems superior to that of human plurality democracies (§2). For more perspective on how good bees are, note that bees apparently outperform everything in table 4.1.

Random-winner	Plurality	Pl+2	Approval	Condorcet-winner (when $\exists$)	Range
5%	19%	26%	45%	77%	81%

Figure 4.1. Monte Carlo simulation results for 5123-voter 20-candidate "honest-voter random normal" elections:[16] for each named voting method, we tabulate its percentage of "best" (greatest utility-sum) winners. Each voter's "utility value" for the election of each candidate is one of 102460 independent standard normal random numbers. For voting systems based on rank-order ballots (plurality, "Pl+2" plurality-with-subsequent-top-2-runoff, and Condorcet) votes are honest. If 0-100 range voting is used each voter gives the best candidate 100 and the worst 0, with the rest honestly linearly interpolated; under approval voting [3] each voter approves of candidates with utility above midway between the best and worst choices. Here "Condorcet winners" (preferred by pairwise majority vote in a head-to-head election with any opponent) exist only 32% of the time, but *when* they exist, they yield best utility-sum 77% of the time. This particular model in a sense gives a uniform sampling of all mathematically-possible elections; its main goals are simplicity and reproducibility, not political realism. ▲

The reasons behind this galling superiority are

1. The bees (essentially) use the best method tabulated (range voting);
2. The bees have little motivation to vote intentionally dishonestly[17] – in that sense human voters are "too clever";
3. The decision problem the bees face (which involves highly *correlated* randomness) is usually easier than the one faced by the computer simulation's voters (*independent* random normals);
4. Only about 5% or fewer of the bees are "scouts" which discover and inspect potential hive sites, and which vote. Scout bees may be better than average at reconnoitering or judging hives (or both) – Lindauer [16] reported that the scouts were precisely the bees that had been experienced foragers in their previous careers – in which case superior decisions would be expected versus voting by *all* the bees.

The honeybee decision procedure has been studied intensively, and entirely decoded, by Lindauer [16] in work more recently redone with modern video technology by Camazine [4] and Seeley et al. [24][27][26][28][29], then modeled mathematically by Myerscough [20]. But due to subtleties and complexities nobody explicitly recognized the essential equivalence of the bees' process and RV. Our sole contribution will be showing that (simplifying Myerscough's model en route).[18]

The bee decision-making process, and why it is essentially equivalent to range voting.
1. Most bees in the swarm find a branch to hang from in an energy-conserving "beard" formation, then sit there.

2. About 5% or fewer of the bees go scouting (initially $\approx$ 1%). If they find candidate nest sites, they (after a careful hour-long inspection) return to the swarm and "report" (via "dancing" using a known kind of sign language [16]) both (a) where their best site is, and (b) how good they think it is. Better sites get more dance repeats, executed with more vigor.[19]

3. Bees who wish to be scouts observe these reports and fly out to check the alleged sites for themselves (as well as, perhaps, doing their own exploring). Also, bees who already have been scouts can choose to re-explore their own sites or those advertised by others. In all cases they report back as before, but bees re-exploring their old favorites, then re-advertise them with successively fewer dance repetitions each time, and once they reach zero, "reset" to an unbiased state.

[16] Each datapoint from 40000 elections.

[17] Since the queen is the only reproducing bee (aside from male "drones," who are not allowed to vote), all voters are motivated by Darwinian evolution to choose the hive they honestly think most likely to lead to maximum reproductive success for that queen. However, this is not *entirely* correct, since worker bees occasionally lay unfertilized eggs which develop into drones – about 0.1% of drones arise this way. Therefore worker-voter bees have a *slight* greedy personal motivation to vote in a way favoring themselves rather than the colony as a whole (when they differ). But if the other workers recognize activation of a worker's ovaries, then they will attack her and eat her eggs! This policing behavior would not have arisen if the worker-reproduction problem had always been this slight, so we may infer that in the evolutionary past, it was far more common. I.e, we know that bee range-voting *evolved* in a situation in which strategic voting, or at least the motivation for it, was far *commoner* than now. Also, even in the current situation, a bee-voter who felt it had superior judgement, would be motivated to strategically exaggerate votes.

[18] The relocation-decision process of *stingless* bees has also been observed, but less carefully. A difficulty: while honeybees communicate using a dance "sign language" which has been videotaped and fully decoded, stingless bees instead use inscrutable olfactory (and perhaps auditory?) communication. Hence whatever "voting process" the stingless bees use is not presently understood. Indeed, I see no strong evidence it is even always *single winner* at all – e.g. perhaps if two sufficiently good alternatives exist, the bees generate two new queens for the occasion and build *two* new hives? It is commonly thought that the honeybees did *not* evolve from stingless bees, but rather independently from some non-eusocial variety, and that any common traits in these two bee types arose from *con*vergent evolution. This conjecture should be confirmable via DNA evidence and if true would make it very interesting to determine what collective decision procedure the stingless bees *independently* evolved.

[19] Dances for housing reports use the same sign-language as dances for food-location reporting, but the housing dances are longer. The fact that humans understand the bee dance language has been confirmed both by traveling to the site the bees were "talking" about, *before* the bees themselves arrive; and also by initiating conversations with the aid of robotic "bee" puppets.

4. So after some time, multiple bee scout "factions" emerge, each advertising different potential nest sites.

Comment: So far, what we have described is a vote-casting process very much like range voting with "intentional blanks" (Xs)[20] allowed: the scout bees cast numerical votes for each candidate, with higher numbers (longer dances) better, and with X's cast for unexamined sites. (And these numbers indeed lie within some range, since there is an upper limit to how long and vigorously a bee can dance.) But, so far, we have only described the vote *casting* part of the process – and not yet how these votes get *combined* to yield an election winner. RV's combining process is numerical averaging. But is averaging beyond bee mental capacity? Averaging involves counting, addition, and division; then picking the winner with the *greatest* average, which involves understanding "greater" and "lesser"; and then, even assuming some bee can do all that, the result must be *communicated* to all the others, and reliably, i.e. they must be sure such communication is genuine and not from a mentally-deranged bee who made a calculational or information-transmitting error. Difficult. So what do the bees do?

5. The bees use an ingenious process which effectively does allow them to reach a consensus on the site with highest-average-vote – RV's goal – *without* any requirement that any individual bee be able to count, add, divide, or compare numbers, and further, the process is designed to be highly *robust* to the presence of occasional mentally-deranged bees! Human designers of "robust parallel algorithms" might well learn lessons from the bees' method.

The process works like compound interest in finance: thanks to the properties of exponential growth, when two bank accounts get different rates of compound interest, the one with the greater interest rate eventually becomes hugely larger (even if initially smaller). The bee-scouts for the best nesting site dance longer and more vigorously for it. The chance a new scout is going to check out a site, is N times bigger if there is N times more dancing for it. That means the exponential multiplication factor, i.e. the number-of-new-scouts generated by seeing the dance of one old scout, is greatest for the best site. Even though the number of dancers for site A may initially be smaller than the number for B, if A induces longer average dance length, i.e. higher exponential growth rate, then ultimately the A-scouts will hugely outnumber the B-faction. And note that it is precisely the average total dance-length among all the dancers for a site, that is its exponential growth rate. So we get exactly RV's effect (at least if we wait long enough): the site with greatest average eventually dominates. (This also has the good side-effect of causing the best candidates to get examined by the most scouts, increasing accuracy and sanity.)

6. The bees' process for determining the highest average score is imperfect for several reasons: (a) Although *eventually* the highest growth rate wins, the bees can't wait forever. The swarm only is willing to sit for at most about 2 weeks and, for speed [29], usually terminates well before full consensus. A late-discovered but better site might not have enough time for its higher growth rate to win. (b) If two sites close in quality are discovered at about the same time, then their growth rates might balance exactly enough to prevent consensus. (c) "Random noise" is involved. Humans using exact arithmetic need not suffer these imperfections and hence (in these respects anyhow) could outperform bees.

7. Honeybees have [29] their own version of the commonly adopted "quorum" rule preventing candidates from winning RV elections with too few genuine numerical votes: They refuse to terminate an election until a quorum of at least 10-20 scouts simultaneously appear at the winning site (implying ≈ 150 had inspected it), presumably because otherwise the quality of its evaluation would be too low. To quote [29]: "We suggest that the quorum size is a parameter of the bees' decision-making process that has been tuned by natural selection to provide an optimal balance between speed (favored by a small quorum) and accuracy (favored by a large one)."[21] Two quorums occasionally arise simultaneously, whereupon the two "winning factions" both "proclaim victory" and both try to maneuver the swarm and queen to their site in a "tug of war" [16]. But then the bees soon recognize this failure, the swarm re-settles, and proceedings recommence to seek a *true* consensus.

5 Ants

Almost all ≈ 20000 known bee species are solitary. Of *social* insects, termites, ants, wasps, and bumblebees do *not* vote on hive relocations because a queen (for termites, accompanied by a single king) founds a new community *solo*. Only the eusocial *bees* decide this collectively. But that impression from [14][18][36] was premature:[22] *Damaging* the nest of the tiny eusocial ant *Temnothorax* (formerly *Leptothorax*) *Albipennis* induces its inhabitants to collectively locate new nest site candidates and choose among them before moving there. Their decision procedure [10][23] resembles honeybees', but:

1. Most scout ants inspect more than one candidate site, enabling direct comparison. (Bees: mostly only one.)
2. There typically are only about 100 ants (for this species), versus about 10^4 bees. The ants usually only have a few sites to choose from, versus typically 20-30 for bees.[24]

[20] Only 22% or fewer [28][4] of the scout bees actually examine more than one site (which might seem to be a severe handicap), hence necessarily employ numerous intentional blanks. Bees can and do dance for more than one site, and sometimes switch opinions in the "wrong" direction.

[21] Passino & Seeley [24] also suggested that the "dance decay rate" in step 3 also was tuned by evolution to optimize a speed-accuracy tradeoff, and gave mathematical-modeling evidence that near-optimal values indeed are used.

[22] Hence it might be wise to experiment with termites or wasps to see if they will vote under the right artificially-induced circumstances, too.

[23] All or almost all ants are eusocial. Worker-ants were found in 90 Myr old amber [11] and 140-210 Myr old fossilized ant nests in Utah, Arizona and New Mexico by Hasiotis & Demko [12], the largest of them many cubic meters indicating decade-century long tenancies. Ants are "arguably the greatest success story in the history of terrestrial metazoa" [25], constituting 15-20% of land animal biomass [15]. The largest ant-colonies house millions. So if ever there were a situation where Darwinian evolution might be expected to devise good social methods, ants are it.

[24] This and the preceding mean the ants have a considerably different *political-mileau* than bees, so it is significant they also evolved RV.

3. While bees have a "dance" sign language letting them inform other bees of both the location and the numerical desirability of a possible nest site, the ants can only tell other ants about locations by requesting they follow (or physically carrying them) there, and apparently can't directly indicate a numerical desirability at all, only "yes/no."

In view of the last fact, it might seem impossible for ants to range vote at all. However, ants can recruit others to inspect a site with different *probabilities* and inter-recruitment *delays*. Hence they can still, in a statistical sense averaged over many recruitment acts, convey continuum "site quality" information as an "exponential growth rate." Then (just as for honeybees) each "faction" supporting a site recruits new members at different exponential growths proportional to the average (mod statistical noise) of some site-quality measure. Eventually, the faction pushing the site with greatest average range-vote will dominate, and again, a "quorum" method cuts off debate once enough ants have assembled at the victor site.

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