Alpha-electric and Nuclear-electric rockets

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Abstract — **In an earlier report, I pointed out that a bilayer film, one of whose sides was made of α-emitting radioactive atoms, would act as a "rocket." A 3-stage alpha rocket could reach, e.g., speeds of $0.0025c$ in a few hundred years.**

(1) I now describe a simple variant I call the "alpha-electric rocket." It uses alpha rays to charge a capacitor which powers an ion gun. If it works, it should be able to reach $5\times$ higher speeds than comparable alpha rockets. We describe 1-stage designs reaching $(0.007\text{-}0.012)c$ in 300 years or $(0.005\text{-}0.009)c$ in 6 years. But it may not work, due to charge leakage problems. That question should be highly amenable to experiment.

(2) The "nuclear-electric rocket" (an un-original idea, but re-analysed here) consists of a nuclear chain reactor generating electricity to power an ion gun. With optimistic assumptions, it could achieve speeds of $0.0006c$ in 100 years – but probably various engineering constraints will reduce that speed estimate by a factor of 4. Thus, the nuclear-electric rocket indeed isn't competitive in speed, at least if required to reach that speed in ≤ 100 years.

But speeds of $0.005c$ could be reached by nuclear-electric rockets running at extremely low power levels for 10000's of years.

Nuclear-electric rockets would enjoy great flexibility advantages (i.e. the ability to turn them on and off at will; possible "refueling" with more expellable mass) versus α rockets, but would suffer from much greater complexity and unreliability.

Along the way, we will derive some new kinds of "rocket equations" and invent some new kinds of ion sources and energy conversion devices.

Keywords — Space propulsion, nuclear reactors, low mass critical assemblies, Cockcroft-Walton ion accelerators, energy conversion devices, electrostatic generators, interstellar travel, negative ion sources, rocket equations, ion propulsion.

1 Introduction

I had proposed and analyzed "alpha rockets" (simply: bilayer films of alpha-emitting and inert absorber materials) in [47]. One 3-stage alpha rocket proposal would reach $.0025c$ in a few hundred years. Major limiting factors were thought to be loss of film and fuel atoms by "sputtering," and damage due to micro-meteor impacts. These issues would best be addressed experimentally, but analysis of what data is known suggested that they could both be kept acceptably small.

The plan of the present report is as follows. We'll first (§2) present and derive some "rocket equations" of independent interest.

Then (§3) we'll describe the "alpha-electric" rocket, which may substantially outperform the alpha rocket, but is almost as simple. This is a large spherical capacitor, charged by alpha emissions, and used to accelerate heavy negative ions. The key open question is whether it is possible to overcome crippling charge leakage problems. The available experimental data (§3.3) suggests that, unfortunately, it won't be possible, but there's not enough data for certainty – far more experiments are needed. Also, more reliable (and hopefully also more efficient) sources of negative ions need to be developed to optimize this idea. For that reason we propose (§4.4.3) a new type of ion source – based on field emission of either positive or negative ions from self-sharpening points on the surfaces of molten salts. This is analogous to the known "liquid metal cusp ion source." We also survey previous kinds of ion sources in §4.4.

In the previous report [47], I had argued that the (non-electric) alpha rocket would attain speeds higher than any other currently practicable method of spacecraft propulsion. But my detailed analysis of all the competing methods had a conspicuous absence: the (well known) "nuclear electric rocket." This is a nuclear chain reactor, generating electricity to power an ion gun.

It is fairly easy to convince oneself (§4) that nuclear electric is clearly inferior to alpha-electric (provided the latter's charge leakage problems may be overcome); it has worse performance, worse reliability, and extra complexity. Despite this, the nuclear electric rocket remains interesting. This is because it may be turned on and off; the α-electric charge leakage question is unresolved; and also because people have proposed nuclear-electric propulsion previously. Hence we feel justified in providing an in-depth analysis of it.

That analysis will show (as was intimated in [47]) that, at least in terms of raw speed reachable in 100 years, nuclear electric is indeed inferior to the alpha plan.

Finally, note that all of these plans (and the alpha and alpha electric plans especially) seem very dangerous to launch from Earth because of the risk of a crash and subsequent

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dissemination of the radioisotope.

2 Rocket equations, old and new

Before we start doing physics and engineering analyses, let us discuss the mathematical theory of rockets.

Our goal is to derive the theory of rockets with variable exhaust speed on a fixed energy budget ("new-style rocket equation"), as opposed to the usual theory of rockets with a fixed mass of chemical fuel ("old-style rocket equation"). (For technical reasons, the new style rocket equation turns out to be surprisingly irrelevant to our particular designs, but it is of independent interest.)

We also want to derive the theory of rockets with constant exhaust speed on a fixed energy budget.

Throughout let M be the initial (fully fueled) mass of rocket, and m be the (smaller) final mass. All our analysis will assume Newton's laws (nonrelativistic) and zero gravity.

2.1 Old-style rocket equation

For old style chemical-fuel rockets, if v is the exhaust velocity and V is the final rocket velocity, then

$$\frac{M}{m} = \exp\frac{V}{v} \tag{1}$$

is the "rocket equation." Since the derivation is given in freshman physics books ([31], 9-47), I'll just summarize it concisely:

$$\frac{\mathrm{d}V}{\mathrm{d}m} = \frac{-v}{m}, \quad \frac{\mathrm{d}V}{v} = \frac{-\mathrm{d}m}{m}, \quad \frac{V}{v} = \ln\frac{M}{m}. \tag{2}$$

2.2 Fixed energy budget, constant exhaust speed

Letting the total amount of energy available be E, the exhaust speed (assuming 100% efficiency; if the efficiency is lower, use a smaller value of E) is $v = \sqrt{2E/(M-m)}$. Hence by EQ (1) the final speed will be

$$V = \sqrt{2E/(M-m)}\ln\frac{M}{m}. \tag{3}$$

If E and m are fixed, this is maximized by picking

$$M \approx 4.921553636m, \tag{4}$$

($4.92\ldots = -2/x$ where $e^x x = -2e^{-2}$) causing

$$V \approx 1.138077534\sqrt{E/m}. \tag{5}$$

2.3 Fixed energy budget, variable exhaust speed

Let $V(M,m,E)$ be the largest possible final rocket velocity, achievable by distributing energy E among the expelled mass $M-m$ in the optimal manner (leading to the greatest final velocity V of the payload mass m). Because of the scaling relations

$$V(aM, am, abE) = V(M,m,E)\sqrt{b} \tag{6}$$

we may express the unknown function V in terms of an unknown function F of only *one* argument

$$V(M,m,E) = F(\frac{M}{m})\sqrt{E/m}. \tag{7}$$

Now

Theorem 1 (New style rocket equation)

$$F(x) = \sqrt{2-2/x}, \quad V(M,m,E) = \sqrt{\frac{2E}{m} - \frac{2E}{M}}. \tag{8}$$

Proof. We'll need lemmas 2 and 3 given below. By the scaling relations[1] EQ (6), the old-style rocket equation 1, and in view of the possibility of employing a constant exhaust speed when our rocket mass ratio M/m lies in the interval $(x, x+y)$, using a fraction z, $0<z<1$, of our total energy budget in this range, we see

$$F(x+y) \ge \max_{0<z<1} \ln(\frac{x+y}{x})\sqrt{2z/y} + \sqrt{1-z}F(x). \tag{9}$$

Now apply lemma 2 to get

$$F(x+y) \ge \sqrt{\frac{2}{y}\ln(\frac{x+y}{x})^2 + F(x)^2}. \tag{10}$$

Now, realize that these two inequalities both are asymptotically exact as $y \to 0^+$ with $M/m > 1$ fixed, because dividing our expelled mass into N equal-mass chunks, each of which are expelled at its own fixed velocity, will reach optimality in the limit $N \to \infty$. We may now use the Taylor series in lemma 3 to get

$$F(x) + yF'(x) + O(y^2) = F(x) + \frac{y}{F(x)}x^{-2} + O(y^2) \tag{11}$$

so that $F'(x)F(x) = x^{-2} + O(yF(x))$, i.e., in the limit $y \to 0^+$, $F'(x)F(x) = x^{-2}$. Upon integrating this differential equation using the obvious boundary condition $F(1) = 0$ we get $F(x) = \sqrt{2-2/x}$, Q.E.D.

Lemma 2 (A maximum) *If $P, Q \ge 0$, then*

$$\max_{0<x<1} P\sqrt{x} + Q\sqrt{1-x} = \sqrt{P^2+Q^2} \tag{12}$$

which occurs when $x = P^2/(P^2+Q^2)$.

Lemma 3 (Two Taylor series)

$$\ln(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \cdots; \quad \sqrt{P+z} = \sqrt{P} + \frac{z}{2\sqrt{P}} - \frac{z^2}{8P\sqrt{P}} + \cdots \tag{13}$$

when $|z| < 1$ and $|z| < P$ respectively.

Remarks. (1) Note that the energy efficiency $mV^2/(2E)$ is $1 - m/M$, so that we approach 100% efficiency in the limit $M/m \to \infty$.

(2) The proof above does not require calculus of variations and hence is accessible with nothing beyond elementary calculus. Instead the technique involves solving N ordinary 1D maximization problems for $N \to \infty$.

[1] These relations may be proven by contradiction. Assume the = is really a < or > at some a and b, then reach a contradiction by scaling the energy usage curves appropriately.

3 Alpha electric propulsion

The proposal is simple. Imagine a large spherical balloon, made of (for concreteness) aluminum foil, and coated with an α-emitting radioisotope. It is best if the films are thin enough that the alpha rays are not appreciably absorbed when passing through them (say 10 microns thickness [57])[2].

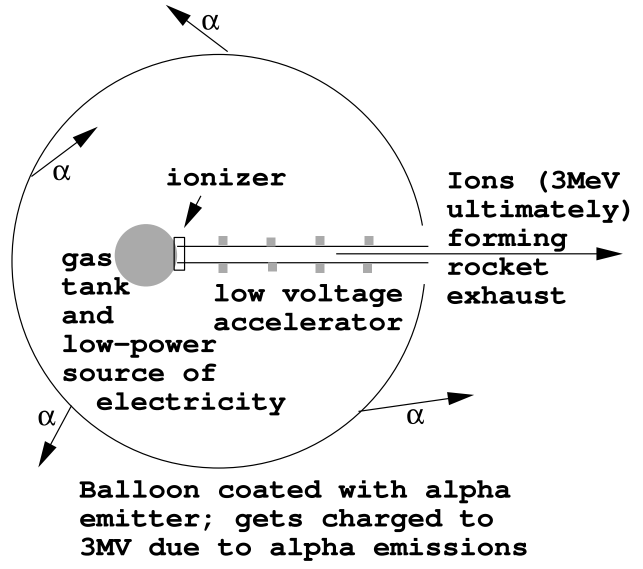

Figure 1: Alpha-electric rocket

Once this balloon is in the vacuum of outer space it may be inflated with some gas. After a brief period, however, the gas may be released and the balloon will nevertheless stay inflated due to Coulombic forces. Specifically, the departing alpha rays, since they carry charge +2, will cause the balloon to acquire a net negative charge. Indeed, the balloon will ultimately acquire a potential, relative to spatial infinity, of 3megavolts (for a radioisotope emitting 6MeV alpha particles).

Now, in addition to the large balloon, our spacecraft will have a tank of expellable mass, a small power source (perhaps a small nuclear reactor or radioisotope thermoelectric generator, cf. §4.2, or a voltage reducing device, cf. §3.6), and an ionizing device (cf. §4.4). These must have a small cross sectional area so that they only intercept (and thus waste) a small fraction of the alpha rays from the balloon. The ionizing device will negatively ionize some molecules and then accelerate them in a rearward directed tube. The departing ions will ultimately reach 3MeV energy of course, but our power source need only provide enough energy to ionize the atoms in the first place and to make sure they are moving in the right direction, e.g. ≤ 25eV per ion. This is tiny; $> 99.99\%$ of the power is being provided (indirectly) by the α-emitting film. (Actually, most present day ion sources have very poor efficiencies, consuming 80eV-9keV per ion produced, see §4.4. But even 9keV per ion is still well below 1%.)

[2]For comparison, I weighed some of the aluminum foil commonly sold in food markets and found it was 29microns thick.

These ions (and any neutral atoms they carry along with them) will constitute the "rocket exhaust." Incidentally, unlike in ion sources for particle accelerators, "hitchhiker" neutral atoms are not necessarily a bad thing because they slow down and scatter our ions. Indeed, we *prefer* it if the mean free path between ion/atom collisions is considerably smaller than the radius of the balloon, so that the hitchhiker atoms and ions will equilibrate their speeds. Our goal is not to get a high energy beam of pure ions; our goal is to get a high momentum rocket exhaust with the right percentage mixture of ions and atoms to get the exhaust speed we want. Unfortunately (as we'll see later §3.8) it seems difficult or impossible to get the small free path lengths necessary to equilibrate speeds, so it will be necessary to try for 100% ionization fraction. (For the same reason, we will want all our ions to have equal charges, here -1.)

3.1 *Why it works better than the plain alpha rocket*

The problem with the original alpha rocket [47] was the fact that the expelled mass (alpha particles) was only a small fraction of the total mass of the rocket. Also, the alpha particles were emitted in all directions, not just the exactly rearward direction. Hence half of them (those emitted in a forward direction) had to be intentionally absorbed and wasted, and even some of the rearward emissions were not very useful for providing thrust. In contrast, we now use the alpha emissions merely as a source of electrical power and not as a source of momentum. Very few are wasted. We get our momentum by expelling a much larger mass (ions and any neutral "hitchhiker" atoms) at lower speeds. The result is greater momentum from the same energy, plus that momentum is directed purely rearward.

3.2 *Film strength requirements*

The capacitance of a spherical balloon of radius R is $C = 4\pi\epsilon_0 R$. If the balloon is charged to voltage V, its self-energy will be $CV^2/2 = 2\pi\epsilon_0 V^2 R$. (For $V =$ 3megavolts and $R =$ 5meters, this is only 2500Joules; hence the initial charge-up time will be small.)

The film will experience an areal tensile stress $S =$d(energy)/d(area) of $S = \epsilon_0 V^2/(4R)$, e.g., for $V =$ 3megavolts and $R =$ 5meters, $S =$ 4.0Joule/meter^2. The 1D tensile stress d(energy)/d$(2\pi R)$ is $\epsilon_0 V^2$, which with $V =$ 3megavolts is 79newtons.

Since the tensile strengths of typical film forming metals are 10-100megaPascals, it is no problem to handle this amount of stress; 10micron thick films have 1-3 orders of magnitude more strength than we require.

3.3 *Electrical losses*

Due to the fact that the insulation (that is, the vacuum) is not perfect, our capacitor will leak (i.e. electrons will leak into the vacuum). This will lead to inefficiency.

It may be that these losses are so severe as to nullify the entire idea.

1. The least understood, and at the same time possibly the most serious, source of charge leakage is ejected "secondary electrons" released every time an alpha or beta particle impacts (or, presumably, emerges from) a solid surface. (Presumably, secondary electrons released on the *inside* of our balloon do not matter, though; with proper design they will eventually be re-absorbed harmlessly.) We'll discuss this below.

2. The electric field at the surface of a sphere of radius R and voltage V is V/R. For $V = 3$megavolts and $R = 5$meters, this is 6kilovolts/cm. The leakage caused by this low a field should be negligible if the sphere is smooth (indeed, experiments show zero leakage across a 1cm vacuum gap between polished steel electrodes even at > 95kV) but if there are tiny pointed uncoated "asperities" on it, they might leak at a much greater rate. Such asperities might be produced as a result of micrometeor impacts.

3. If the ionizer, in addition to producing negative ions, also emits free electrons, these could (in the event they do not recombine with neutral atoms) cause charge leakage. But it should be easy to separate out and recover almost all such electrons with a magnetic field.

4. If the radioactive film also emits beta rays (free electrons, charge -1) then these will be a serious source of leakage. It might be possible to reduce this leakage by using a magnetic field from a loop of superconducting wire to cause some of the betas to circle back numerous times and hopefully be re-absorbed by the film. This magnet idea might indeed be useable to re-absorb all secondary electrons – solving all our problems. However, this idea might actually make the problem worse due to the emission of *more* secondary electrons. To be conservative and/or to keep the analysis simple, we will assume in our analysis below that no magnetic field is present.

Secondary electron emission caused by electron impacts has been heavily studied, but there has been comparatively little attention devoted to the same phenomenon caused by impacts of alpha-particles (or ions generally). Also, presumably, secondaries are also emitted when alpha or beta rays *emerge* from a surface, but that apparently has never been studied.

The number of secondary electrons caused by *electron* impacts can be as large as 900 for materials designed to be ultra-emissive (such as Si doped with Cs) but is in the range 0.45-1.55 for elements – at the energy of the incident electron which maximizes yield [19]. This yield-maximizing energy lies in the range 85-800eV. If the incident electron has substantially larger or smaller energy than this, then the yield of secondaries is small.

It is believed that "field aided emission" rarely has much effect, i.e. the presence of an electric field does not alter the secondary yield by much.

For *alpha particles* and ions generally, the main conclusions of the studies so far [2][3][4][14][15][29] are as follows. Each impact of an alpha particle (or a positive ion in general) onto a metal surface yields some emission of electrons, say Y of them on average. (Y is the "yield.") These electrons almost all have low energies (< 20eV), no matter what the energy of the ion. Y appears to be a sum of two sub-yields $Y = Y_{\rm PE} + Y_{\rm KE}$ due to potential and kinetic energies associated with the ion. The PE component is dominant for incident ion velocities < 50km/second, but the KE component is dominant for velocities > 100km/second, roughly. (The crossover velocity is supposed [2] to be comparable to the Fermi velocity of electrons in the metal.) $Y_{\rm PE}$ is thought [3] to be roughly proportional to the total neutralization energy of the ion, times $[1-\exp(-v_{\rm crit}/v)]$, where v is the ion's velocity. This goes to zero when $v \gg v_{\rm crit} \approx 50$km/second, but goes to a constant when $v \to 0$. The constant depends on the ion and the solid; for low velocity He^+ ions impacting tungsten, Hagstrum [29] found $Y \approx .3$, and for He^{++} ions $Y \approx .72$; however Chapman [15] indicated much smaller Ys (0.01-0.02) in typical scenarios encountered in sputtering by ions. A large number of low energy Ys for various ion-target pairs are mentioned in Carter's excellent survey [14]; the discrepancy above is explained by the fact that He^{++} has exceptionally large low-energy Y. At high ion velocities, $Y_{\rm KE}$ is an increasing function of v which seems to tend toward a constant for large (but nonrelativistic) v. This constant depends on the ion and the solid. For 3000km/second ions (H^+, D^+ He^+, Ar^+) impacting clean metals (Li,Al,Cu,Ag,Au), it seems [4] that $1.1 \le Y \le 2.8$. My theoretical grounds for believing $Y_{\rm KE}$ tends to a constant at high ion energies are that many theoretical and experimental studies [57] show that ions give up a constant amount of energy per unit length (independent of their speed) as they fly through solids, and so we expect a constant number of electrons within 1 free path of the surface to acquire "escape velocity" (which corresponds as an energy to the "work function" of that metal). For typical metals, the free path is around 5nm. Experimental evidence [14] suggests the electrons come from a layer of thickness ≈ 12nm. There have also been related theories proposed [14][25][49] involving Y actually decreasing to 0 at high ion speeds v proportionally to $1/v$ or $(\ln v)/v$. These theories seem to agree well with data for for H^+ ions impacting tungsten [22], for which a Y maximum occurs at about 125KeV with $Y \approx 1.65$, followed by a Y-decrease to about $Y \approx 1$ at about 2MeV. The theories of [25][49] predict asymptotic independence on work function and conductivity at high ion energies, which seems contrary to observations [4][14]. Sternglass [49] also predicts Y should be considerably higher for insulators than for metals.

There have been no studies that I know of about secondaries resulting from ion impacts on *insulating* materials, but [4] mentioned that an oxide coating formed on their aluminum sample due to air exposure, and caused $Y \approx 7$, but as the oxide eroded due to continued exposure to 30keV Ar^+ ions, Y decreased, at first rapidly to about 4, then slowly to its final value (which was the same as the value for clean Al metal) of 1.7. Sorbed gases tend to decrease Y [22][30] and the (100) and (110) faces of copper crystals have very different Y [14][38]. All this suggests that yields are very affected by surface conditions.

All of the (ion, metal) combinations studied in [4] obeyed Ag>Au<Al<Cu<Li (where "$A < B$" means higher yield for metal B than for metal A for same ions at same impact energies). This is entirely compatible with the fact that the "work functions" (energies required to eject an electron) of these metals obey Li<Al<Cu<Au>Ag *except* note that the pair {Al, Cu} is anomalous. Perhaps this is because Al is a substantially worse conductor than Ag, Au, and Cu, i.e. has a substantially shorter electron free path. (Li is also a poor conductor, but its work function of 2.93eV is so much smaller than the other metals's 4-6eV that this might not matter.)

That suggests that we want (to get small secondary electron yield) a metal (or metal alloy) with large work function, but small electrical conductivity. Perhaps amorphous metal alloys such as "metglas" would have these properties. I do not know what effect magnetic alloys would have, nor superconductors, nor what should be said about nonmetals. Certain Cu-Be compounds have very high Y (≈ 10) for 20keV ions [14]; why?

Let us conclude. If $Y \geq 2$ "secondary" electrons are ejected per alpha ray, our whole idea will be completely nullified. For high efficiency we need to keep Y well below 2, e.g. 0.05. Not enough experiments have been done, as yet, to tell whether this goal is feasible – only about 6 kinds of surfaces have been studied so far. Since all of those surfaces had high energy ion-impact yields $1 \leq Y \leq 7$, alpha-electric rockets quite likely are in trouble, but I don't think enough evidence is in yet to judge. For example, no intentional efforts seem to have been made yet to produce low-Y materials.

Experiments to address this question should be quite easy to perform[3]. Simply take an alpha emitter, place it in vacuum, and connect it to an electrometer. See how quickly charge builds up on it, versus the (known) number of alpha rays emitted. Try various kinds of surface films and treatments – which minimize the secondary yield?

We shall assume in the below (in order to proceed, as opposed to just quitting immediately) that it will be feasible to achieve small electron yields $Y \ll 2$ from alpha ray surface impacts and emergences.

3.4 Other worries

Issues of sputtering, other kinds of radiation damage, drag due to matter in the (imperfect) vacuum of space, and damage due to impacts with that matter, have already been discussed in [47]. The first two of these (as was pointed out in [47]) deserve further experimental investigation.

We are assuming that our ionizer can last for as long as required, whereas in fact present-day ion sources require cleaning or maintenance every 3000-10000 hours (§4.5).

[3] Much easier than the experiments I cited.

3.5 Extra thrust

If the alpha-emitter is layered more thickly on the back side, and more thinly on the front side, of the balloon, then (since no film is *perfectly* transparent to alpha rays) the alpha rays will also contribute to the thrust in the same manner as the original alpha rocket [47], leading to slightly higher efficiencies than otherwise. We can also get the same effect by locating the alpha-opaque tank, etc., *non-centrally* inside the balloon (i.e., more toward its front side).

3.6 Nuclear battery

It is also possible to view the alpha-emitter coated balloon as a DC power source (essentially, a highly efficient "nuclear powered battery," or a capacitor that keeps getting recharged) at 3megavolts and low current. (The other terminal of the battery would be a sharp point or heated filament to emit electrons; or the negative-ion source for our rocket exhaust, itself.) This battery may be useful for purposes other than propulsion. For example, it might be possible to use this source to power the ionizer, or to power instruments even on a non-accelerating spacecraft.

The main difficulty in using this power source is the need to convert down to lower voltages at higher currents to be compatible with most power-consuming devices. Such conversion could be accomplished, for example, with a Cockcroft-Walton voltage multiplier [17][36] working in reverse as a voltage divider, or with a Van de Graaff generator in reverse (that is, a moving belt, or rigid rotor, carries charge off the balloon; this rotational energy is converted to low voltage electrical energy with a mechanical generator).

3.6.1 Cyclotron in reverse (this section may be skipped)

Another interesting kind of "nuclear battery," (which I do not have the space to analyze fully here, but mention because it is a new invention, conceivably with some use as a future efficient power source) is the "cyclotron in reverse." This consists of a mm-thick film of β-emitting isotope. The expelled electrons go into a vacuum region containing a static magnetic field B (from a permanent magnet or superconducting coil) then describe helical paths with angular frequency $\omega = eB/m_e$. Each member of a pair of D-shaped electrodes surrounding the vacuum chamber will be cyclically nearer and further from the electrons and hence an AC voltage will be developed between them. This may be transformed to any desired voltage level with an RF transformer and bled off to power some external load.

The orbiting electrons gradually lose energy and spiral into the center of the chamber, at which point they could be removed. With $B = 0.1$ tesla, $(2\pi)^{-1}\omega \approx 2.8$GHz. With 315keV electrons (^{60}Co) the radius of the circle would be 1.9cm. Since ^{60}Co has a halflife of 5.3 years, 1 gram of it will emit about 6×10^{13} electrons per second, corresponding to about 3 watts at a current of 10 microamperes.

The voltage difference between the Dees will be of order $q_e/(\epsilon_0 R)$ per electron, where R is the characteristic size of the Dees. This has to be multiplied by a factor depending upon their precise geometry which for our present crude purposes we shall regard as 1. (An equivalent formula is that the voltage difference is q_e/C where C is the capacitance between the Dees.) All the calculations in the remainder of this paragraph therefore may be off by a factor of 10. If $R = 2$cm ($C = 16$picoFarad) this is of order 1 microvolt per electron. Assuming all emitted electrons from the 1 gram ^{60}Co source end up in the cyclotron, and each electron makes 3000 orbits before it is removed, there will be about 8×10^7 electrons in the system at any time, leading to an AC voltage of order 50 volts across the Dees. Since $3000 \times 2 \times 50$ volts is about 315 kvolts, this assumption (of 3000 orbits) works.

It is necessary to modulate the β-source (perhaps using a resonant EM mode to divert electrons to various entrances of the cyclotron, or various cyclotrons, at different times). Indeed with proper design, such a modulating mode might occur naturally due to positive feedback. It also will be necessary to focus the electrons (initially emitted uniformly in angle) into a narrow range of angles. Well known (nowadays) cyclotron design ideas of azimuthal B-field variation could be used to try to handle relativistic electrons. Computers would undoubtedly have to be used extensively to get a good design.

3.7 *Pessimistic performance estimate*

Assume the balloon film weighs 100 kg (10 micron thick film, density 8grams/cm^3, and 10 meter radius), and half its mass is radioisotope; the other half is inert. Assume the small power source, ionizer, and the tank (when empty) weigh 50kg total. The total mass is then $m = 150$kg; *plus* let us assume the expellable mass is 3.92 times this, i.e. 588.2kg, so $M = 738.2$kg. The choice "3.92" is due to EQ (4). Assume the total available energy from the radioisotope is $E = 5.2 \times 10^{14}$Joules. That arises from assuming each radioactive nucleus emits 8 alpha rays (each > 5.4MeV) and 3 beta rays (all this is the case for ^{240}Cm, with halflife 27 days, but whose total decay chain has summed halflife 74 years) and assuming that with N alpha's and n beta's emitted, the total energy extractable to charge the balloon is $E = (N - n/2)4$MeV. In this formula we decreased 5.4MeV to 4MeV to account for inefficiency caused by partial absorption of the alphas, and/or the fact that the distribution of exhaust velocities will not be exactly a delta function, due to the fact that the exhaust gas may not be dense enough for enough time for collisions between the ions and the neutral atoms in the exhaust to fully equilibrate all their velocities.

In that case EQ (5) tells us that the final speed will be 2100km/second$= 0.0071c$. In comparison, an alpha rocket using the same isotope would only reach about 1/5 this speed.

If instead of ^{240}Cm we used ^{229}Th (which is in the tail of the ^{240}Cm decay chain and emits 5 alphas, each > 5.5MeV, and 3 betas) we would reach 1600km/second$= .0054c$. This would have the advantage that we would only have to wait a few years to acquire most of this speed, since ^{229}Th has halflife 1.91 years.

3.8 *The exhaust*

The mean free path of xenon gas atoms at .001mm of mercury pressure (about 10^{-6} atmospheres, or about 10 pascal) at room temperature, is 2.6cm. For Helium gas atoms at the same pressure and temperature, it's 13.3cm. Hence, for a balloon with radius 10 meters, our assumption of small free path inside the exhaust gas (small compared to 10 meters, that is) will be satisfied for gas pressures above about 100millipascals at room temperature. The thrust levels being proposed are in the neighborhood of .1-100 Newtons, which (for a cross sectional area of 400meter^2) corresponds to a pressure (thrust per unit area) of about .2-200 millipascals. Hence we can see immediately that our desire for a short free path compared to the balloon size, is *not* easy to satisfy. If the beam width of the exhaust could be kept small (< 1 cm radius) then that should be adequate at thrust levels exceeding about 1 Newton, but probably this narrow a beam is infeasible. Hence, it will probably instead be necessary in practice to try to get high ionization fractions in the exhaust (optimally, approaching 100%) instead of relying on a short mean free path the equilibrate ion and neutral velocities in an ion/neutral mixture.

In that case we are going to need negative ions of mass about 257amu (presuming they are singly charged; it is presumably almost impossible to get multiply charged *negative* ions) to get the optimal mass balance. That would be possible most simply using depleted uranium (238amu), but one could also consider using ionized molecules made of lighter elements. Unfortunately, present day ion sources (cf. §4.4) tend to feature ionization fraction $f < 1$, in which case all our analysis should be altered by multiplying all velocities by f. (Similarly, e.g., if the yield of secondary electrons emitted during an alpha particle's emergence from the balloon's surface is Y, which reduces the effective charge of each alpha particle from 2 to $2-Y$, then all velocities should be multiplied by $\sqrt{1 - .5Y}$.)

We have nevertheless assumed that $f = 1$, because I believe there will be no obstacle to achieving ionization fractions very close to 100%. For example, with ions made from atoms of a solid such as uranium, neutral atoms could be diverted out of the beam (using electrostatic or magnetic deflection as in a mass spectrometer) onto a cold solid surface (no problem if we are in the limit of large free path), and recovered. (Accidentally created electrons could and should similarly be removed from the beam.) Such recovery would cost energy, but since as we've pointed out already, these energy scales are $< 1\%$ of the energy being used for thrust, this is not a grave concern. A more serious concern is simply the extra complexity and unreliability inherent in devices to accomplish this mass recovery[4].

[4] Of course, separation of ions from uncharged particles is done every day in hundreds of accelerators and mass spectrometers around the world. I am not proposing a new idea here. The only objection to this idea, then, could be on engineering grounds of complexity or unreliability.

3.9 Optimistic performance estimate

If (now thinking optimistically) we were able to make the film be 100% radioisotope, without need for any inert reinforcing material, and could parasite our "nuclear battery" for power to run the ionizer, could keep the ionizer mass small, and could avoid need for a heavy pressurized gas (or cryogenic liquid) tank by using, instead of a gas, a liquid or a solid (e.g. reel of wire), so that the total non-expelled mass was only 110kg, 100kg of which was radioactive, then we could get 1.65× greater speeds, e.g. the ^{240}Cm rocket would attain $0.012c$ and the ^{229}Th rocket $0.0089c$. With this configuration, the optimal ion mass would be 94amu rather than 257amu.

3.10 Staging

Both alpha rockets and alpha-electric rockets may be "staged" to reach 2-3 times greater ultimate speeds (the smaller-massed later stages are powered by longer-lived isotopes).

4 Nuclear electric propulsion

There are 6 main components in a nuclear-electric rocket:

1. Tank of expellable mass.
2. Nuclear fission chain reactor (generates heat).
3. Radiator (gets rid of waste heat).
4. Ionizer (converts expellable mass atoms into ions)
5. Ion accelerator (expels the exhaust)
6. Energy conversion device (converts heat energy flowing from the nuclear reactor to the radiator into electrical energy, and/or energy in whatever form the accelerator consumes)

and each will be discussed in a separate section.

Finally we'll make a rough design of a combined system and estimate its performance, getting $(.0002\text{-}.0006)c$.

One can see immediately, without need for detailed analysis, that nuclear electric rockets have no hope of reaching the same speeds as alpha electric. Why is this? Well, nuclear fission releases about 200MeV of extractable energy per nucleus, whereas 8 alpha emissions will only release about 50MeV. So to compete, the conversions of power from nuclear to heat to electricity to high voltage electricity must have a total efficiency of $> 15\%$ or so (since in the alpha electric rocket, nuclear energy is converted directly to high voltage electricity with an efficiency of $> 60\%$). That might indeed be achievable but then the nuclear electric rocket would only reach the same speed if the radiator, accelerator, and energy convertor weigh zero – or more precisely, mass much less than the mass of the fissionable isotope that the energy comes from. (If we postulate 30% efficiency, then they must mass in total less than the mass of the fissionable isotope.) These mass constraints are ridiculous – they in fact will weigh *much* more. The greatness of the alpha electric plan, which makes it so hard to beat[5], is its simplicity: the "radiator," the power source, the charge storage capacitor, the energy conversion devices, and the accelerator are all the same object, which is just a balloon!

It is nevertheless of interest to analyze the true performance of the nuclear electric plan.

It appears that components 3, 5 and 6 are the performance limiting ones, and our mass and speed numbers (even the more conservative ones) are somewhat speculative because they depend on estimated future technological capabilities in the last two areas. If one is willing to expend energy at a very low rate, waiting an extremely long time to reach high speeds (many thousands of years), then the radiator and energy convertor no longer are performance limiting, and speeds of $0.005c$ become possible. On the other hand, the decision to employ such low power largely sacrifices the greatest advantage nuclear-electric rockets have versus the alpha rockets – the ability to turn them off and on, where "on" means high power.

4.1 Tank of expellable mass

Suppose we have a tank of 200kg of xenon (atomic mass 131.3 amu) i.e. 9.2×10^{26} xenon atoms. Assume we want to use up that xenon in 100 years. That is a rate of 2.9×10^{17} xenon atoms/second. Assuming each is charged to +1, that is 47 millamperes of ion current. Assuming we accelerate the xenon ions to average energy 1MeV, that is 47 kilowatts of beam power (voltage×current) for 100 years, which is 1.47×10^{14}Joules. If not all of the xenon atoms in the exhaust will be ionized, then reaching the same exhaust speeds would require more voltage (or we could use lighter atoms than xenon, such as sodium wire, or liquid mercury).

4.2 Nuclear reactors with small masses and large energy/mass ratios

Most people think of nuclear reactors as objects weighing many tons at least. It is not commonly realized (and this apparently has not been taken advantage of in U.S. and Russian space programs so far) that it is possible to make critical assemblies weighing < 10kg. I will discuss various alternatives in considerable detail to make it clear gradually what the optimal reactor would be like (^{233}U dissolved in 10 liters of pressurized water in a spherical tank). I want to make it clear ahead of time that these sort of reactors have been built, are commercially available, and have many years of operating experience [56].

Actually, there are 4 different interesting and related challenges:

1. Creating critical assemblies of the smallest possible total mass;

[5] Another advantage: the "accelerator" in the alpha-electric plan is "unipolar," i.e. a high voltage sphere, with "ground" being at spatial infinity. But in the nuclear-electric plan, the accelerator would have to be "bipolar," that is, the spacecraft would have to provide its own "ground," i.e. would have to charge one electrode to high positive and one to high negative voltage. One of these two electrodes then would be useless deadweight.

2. Smallest possible mass of the (expensive) fissionable isotope, but allowed to use a large mass of other materials

3. Largest possible ratio of energy generated to total mass.

4. Lowest possible concentration of fissile isotope in inert moderating matter.

Challenge #3 is the one most relevant for us. The best answers to the first three of these problems all seem to involve an intimate mixture (a slurry or solution) of some ^{233}U-containing chemical in a hydrogen-rich moderator material[6].

The best possible nuclear fuel would emit a large number of neutrons per neutron absorbed (allowing small critical masses), would have a long "shelf life" (halflife in the absence of thermal neutrons), and would have a high fission energy release per unit mass.

Among the fissionable isotopes ^{233}U, ^{235}U, ^{239}Pu and ^{241}Pu: ^{233}U has the highest ratio (2.29) of neutrons emitted per thermal neutron absorbed, and hence yields the smallest critical masses; ^{233}U also has the highest probability of actually fissioning (as opposed to just capturing the neutron) if hit by a thermal neutron, which causes it to have the highest effectively extractable energy per unit mass, about 7.6×10^{13}Joules/kg. ^{233}U has a fully satisfactory "shelf" halflife of 160kYears, also. Unfortunately, ^{233}U presently (1999 dollars) costs about $6950/gram, while ^{235}U is only about $57/gram. However, that economic comparison may reverse in the future as breeder reactors become more common. (^{233}U is produced by "breeding" from Thorium.)

H	.332b	^{16}O	.19mb	Fe	2.56b
D	.52mb	O	.28mb	^{40}Zr	.18b
T	$< 6\mu$b	^{18}O	.16mb	Cd	2450b
^{10}B	3837b	F	9.5mb	^{149}Sm	41kb
^{11}B	5mb	Al	1.6b	Gd	49kb
C	3.5mb	S	.49b	Pt	10b
^{15}N	.04mb	Cr	3.1b	Pb	.17b

Table 1: Absorption cross sections for thermal neutrons at 300K. If atomic mass number is not given, natural isotopic abundance mixture is intended. One "barn" (b) is 10^{-28}meter^2.

The chemical forms of uranium that seem most interesting for us (aside from the metal itself, with density 18.9grams/cm^3) are the hydride UH_3, (a pyrophoric black powder, density 10.95, that forms at 250-300C from H_2 and U, but decomposes at 440C at 1atm pressure; at 580atm, is stable at least up to its melting point 1050C), Uranyl Sulfate UO_2SO_4, and Uranyl Fluoride UO_2F_2. The latter two compounds are soluble in water.

The best possible moderator would have a large cross section for scattering neutrons, leading to a small "RMS slowdown length" in which fast neutrons thermalize, but a small cross section for absorbing thermal neutrons. Since the moderator represents "deadweight," we also wish[7] to have a very small total mass of moderator in comparison with the mass of the ^{233}U fuel. For our purposes absorption of neutrons by the moderator is going to be very small in comparison to absorption of neutrons by the fuel atoms (because we are operating in a regime of high fuel:moderator mass ratio), so the only thing that matters, in order to keep the physical size, and hence the mass, small, is minimizing the RMS slowdown length. That is best accomplished [26] by using a moderator material with a maximum number of light hydrogen atoms per liter.

Table 2 lists materials with high hydrogen densities.

Of these, (2,2,3,3)-tetramethyl butane (CAS #594-82-1, Beilstein 4-01-00-00447) is quite remarkable: it is by far the most thermodynamically stable octane C_8H_{18}, and at the same time the densest, and it has a remarkably high melting point of 100.7C, remarkably near its boiling point 106.4C.

None of the small alkanes, however, gives as large a hydrogen density as polyethylene $(CH_2)_n$. "Linear high density" polyethylene, polymerized with Ziegler-Natta catalysts, is better than the lower density polyethylene synthesized with the older, high pressure, process.

Beryllium hydride [10][46] seems to be the best moderator material of all. But it is conceivable that polymerizing tetramethyl ethylene would yield a polymer $(C_3H_6)_n$ with a record high hydrogen density (since locally it looks like (2,2,3,3)-tetramethyl butane, but globally like linear polyethylene).

It is best to use a H:U atom ratio of 10-20:1 initially (after the U fuel is consumed, this ratio will rise). We want to use as little H (and moderator) as possible, of course, to decrease "deadweight." However, using less H than this would cause the flight length between neutron scatterings to start becoming substantially larger than the flight length between absorptions of thermal neutrons by Uranium nuclei, which would render the moderator pointless and cause the critical mass to depend considerably on the amount of U fuel remaining, whereas we prefer it if reactor behavior is largely independent of this.

These last two moderator materials in table 2 would lead (when molded with UH$_3$ powder) to the smallest-mass critical assemblies. However, the resulting lump of plastic will chemically decompose, emitting, e.g., hydrogen and methane gas, during exposure to radiation, and *there is no easy way to put its molecules back together again.* (BeH$_2$ does not form from its elements and must be synthesized indirectly.) Hence they seem unsuitable for *prolonged* operation.

[6] The best answer to the last challenge seems to be ^{233}U in ^{15}Nitrogen at a U:N atomic ratio of about $1{:}1.5 \times 10^7$; even smaller concentrations are possible if tritium is permitted.

[7] Actually, the best possible energy/mass ratio would seem to arise by dispensing with the moderator entirely, and making a fast-neutron critical assembly using solid metal ^{233}U, ^{235}U, or ^{239}Pu. The problem with that idea is that such assemblies would have to be controlled, to prevent them from exploding, at gigahertz frequencies. I do not know how to accomplish that. It is comparatively easy to control reactors based on slow neutrons, and indeed the ones we propose will be largely self-controlling because of negative temperature coefficients.

stuff	density	melt	boil	crit.temp.	crit.press.	molec.wt.	H
	grams/cm^3	K	K	Kelvin	MPa	amu	moles/liter
H_2O	1	273.1	373.1	647.1	22.06	18.02	111.03
$(\ell)H_2$(15K)	.152	13.8	20.3	33.0	1.293	2.02	75.48
$(\ell)NH_3$(239K)	.6819	195.4	239.8	405.5	11.35	17.03	120.01
$(\ell)CH_4$(100K)	.439	90.7	111.6	190.5	4.604	16.04	109.48
$(\ell)C_2H_6$(100K)	.641	90.3	184.5	305.4	4.88	30.07	127.92
$(\ell)C_3H_8$(100K)	.719	85.5	231.0	369.8	4.250	44.10	130.36
n-C_4H_{10}	.573	134.9	272.6	425.1	3.784	58.12	98.59
n-C_5H_{12}	.6262	143.4	309.2	469.7	3.364	72.15	104.15
n-C_6H_{14}	.6548	177.8	341.9	507.7	3.010	86.18	104.91
*-C_6H_{14}	.6616	144.3	331.1	500.0	3.131	86.18	107.48
n-C_7H_{16}	.683	182.5	371.6	540.3	2.756	100.17	109.03
*-C_7H_{16}	.6982	154.5	366.6	540.7	2.891	100.17	111.52
n-C_8H_{18}	.6986	216.3	398.8	568.9	2.493	114.23	96.82
*-C_8H_{18}	.8242	373.8	379.6	567.8	2.87	114.23	129.87
n-C_9H_{20}	.7176	219.6	424.0	594.6	2.288	128.26	111.90
*-C_9H_{20}	.7547	171.0	414.7	607.7	2.716	128.26	117.68
n-$C_{10}H_{22}$	.7300	243.4	447.3	617.7	2.104	142.29	112.87
*-$C_{10}H_{22}$	.7789		443.1			142.29	120.44
n-$C_{16}H_{34}$	.8129	291.3	560.0	722		226.45	122.05
LD polyethylene	.91-.93	382					129.8-132.7
HD polyethylene	.941-.965	463					134.2-137.7
BeH_2	.755					11.028	136.9

Table 2: Materials with high hydrogen density. Densities are given at room temperature and pressure unless otherwise stated. The *'s indicate the *best* (i.e. densest) isomer, which for C_6H_{14}, C_7H_{16}, C_8H_{18}, C_9H_{20}, and $C_{10}H_{22}$ are respectively (2,3)-dimethyl butane, 3-ethyl pentane, (2,2,3,3)-tetramethyl butane, (2,3,3,4)-tetramethyl pentane, and (3,3,4,4)-tetramethyl hexane; n indicates the straight-chain isomer.

For that purpose, I would recommend ordinary water H_2O and operation at temperatures of 250-350C at corresponding pressures 4-16.5megaPascals (the critical temperature and pressure of water are 374C and 22MPa [19]).

Although water has about a 25% larger RMS slowdown length (5.2cm), and hence will lead to critical masses about $1.25^3 \approx 1.95$ times larger, than the optimal moderators, it has several advantages.

First, after exposure to radiation, water decomposes into H_2 and O_2 gases and with formation of H_2O_2. (For example, at 50kW thermal output, a reactor made of uranium dissolved in 12 liters of water evolved about 12 liters/minute of H_2 and O_2 gases [56].) *These chemical changes are easy to reverse.* H_2O_2 is decomposed into water and O_2 gas by numerous catalysts, the best of which for our purposes is probably Pb ions (since they have a low cross section for thermal neutrons: 0.17barns) [44]. (Fe and Cr ions also have a catalytic effect and small neutron cross sections, although they are each about 10 times worse than Pb in both respects.) H_2 and O_2 gas are readily and harmlessly catalytically recombined using such catalysts as Pt in Al_2O_3 [56].

Second, water is a liquid, allowing the fuel slurry or solute to be mixed and hence kept homogeneous. (In contrast, polyethylene and BeH_2 are solids.) It has a high heat of vaporization too (2260joule/gram).

On the other hand, water expands when it freezes. Also, cold water can't dissolve uranium salts at as high concentrations as we would like [37][45][39], see table 3.

temp	moles	temp	moles
-38.5C	3.82	181C	8.66
30.0C	4.135	299.9C	12.30
93.5C	5.06	374C	7.02

Table 3: Solubilities of UO_2SO_4 in water, moles per kg of water.

Some of these saturation concentrations may be boosted by perhaps 20% by acidifying the water with H_2SO_4. Therefore there would be no solubility problems at typical operating temperatures, but at low temperatures, there would be risk of freezing damage or of unintentional formation of critical masses in precipitating crystals. These risks should be eliminable with correct design, but must be borne in mind.

Data on neutron cross sections and slowdown lengths may be found in [26]. Typical critical masses and sizes [13][28] are given in table 4.

The moderated critical masses quoted are at 25C. As temperatures are raised from 25-95C, the ^{235}U and ^{233}U critical concentrations increase by about 8% and 10% respectively, due to the lower cross sections of these isotopes for neutron absorption and neutron-induced fission at higher neutron temperatures. Such "negative tempco" effects are very useful for keeping one's reactor stable. One could also use this effect in reverse to try to get extremely low critical masses

^{233}U metal sphere in air. 16kg? (11.8cm diam?)
^{235}U metal sphere in air. 52kg (17.5cm diam)
^{239}Pu metal sphere in air. 21Kg (12.7cm diam)
$^{235}U+H_2O$ sphere, 32cm diam; 2.13kg U + 17kg H_2O
$^{233}U+H_2O$ sphere, 32cm diam; 1.15kg U + 17kg H_2O
$^{235}U+H_2O$ reflectorized sphere, 6.5 liters: 3.0kg U
$^{233}U+H_2O$ reflectorized sphere, 8 liters: 0.6kg U
$^{239}Pu+H_2O$ ref'd sphere, 13 liters: 0.9kg Pu.

Table 4: Critical masses at 25C. ("Reflectorized" spheres are surrounded by some combination of very thick stainless steel and water.)

at low temperatures (which might be useful for something, such as very high reliability and low power reactors for space applications)[8].

Extensive operating experience has been gained with "water boiler" reactors of this type [56]. For example, in low-Cl water (< 5ppm Chlorine), corrosion of stainless steel coupons placed in such reactors was < 25microns per year (and welds were not corroded at a different rate), and after 3 years of running, the stainless steel containment vessel was found to be eroded < .5microns per year. This latter corrosion bound is fully acceptable even for 1000 year runtime.

As is well known [26], during operation the fission products ^{135}Xe and ^{149}Sm (and others) will build up and cause "poisoning" of the reactor, multiplying the effective fission "reactivity" of the uranium fuel by a factor $\geq .92$ in steady state operation. The multiplier can be as severe as .65 after a pessimally long wait after shutdown. This is for reactors based on ^{235}U; poisoning is less understood for ^{233}U-based reactors. Extra reactivity has to be designed into the reactor to compensate for this. Similarly, some sort of "control" has to be designed in to allow preventing excess reactivity and too amplified a chain reaction and too high temperatures.

4.3 Radiator

Getting rid of waste heat in spacecraft is considerably more difficult than getting rid of it on, say, an aircraft, because we must use a radiator rather than convective cooling. However, radiators have the advantage that they are easy to analyze.

If the spacecraft is not allowed to exceed temperature T, then getting rid of an amount of heat power P using a radiator will require the area A of the radiator to obey

$$A \geq \frac{P}{\sigma_{SB} T^4} \tag{14}$$

where $\sigma_{SB} \approx 5.67 \times 10^{-8}$second^{-1}Kelvin^{-4}kg is the Stefan-Boltzmann constant. In particular, if $T = 350$Kelvin and $P = 1$megawatt, then $A \geq 1175.2$meter^2. Assuming the radiator is made of 1mm-thick aluminum sheet (mass 2.7kg per square meter; when one considers that this mass must include, e.g. pipes – resistant to micrometeor impact – to circulate coolant, the coolant, and the pump for that coolant, this estimate seems conservative) and (to get twice as much area per unit mass) assuming the radiator is 2-sided, then the radiator will have to mass at least 1587kg. If the allowable temperature is 500Kelvin, the radiator would only have to weigh 381.2kg. Note that it is possible to design pumps based on electrical principles which have no solid moving parts. A different idea involves a radiator which is a continuously moving flexible belt with one hot and one cold roller. This avoids the need for liquid coolant.

Either way, we can see that the radiator is a tremendous performance limitation, since its dead weight seems to greatly exceed the mass of the nuclear reactor generating that heat.

4.4 Ion source

One of the few advantages nuclear-electric has versus alpha-electric, is that it can use an ion source for *positive* ions, whereas with alpha-electric, it is essential to expel negative ions. At present, sources of negative ions are considerably less technologically developed and also tend to be less efficient.

There are several interesting kinds of ion sources[9] suitable for spacecraft propulsion:

1. Those based on electronic interactions between gas atoms (to be ionized) and surfaces,
2. Electron discharges in gases,
3. Field emission from sharp points created by electrostatic means on liquid metal surfaces.

4.4.1 Gas discharge sources

Gas discharge sources [5][7][11][23][24][27][34][41][50][51] involve a continuous direct current discharge through a gas. Radio frequency AC discharges are also possible and can lead to advantages in terms of simplicity of design, although they tend to have a slightly lower efficiencies due to, e.g., eddy currents. The discharge often is "pinched" or elongated by the action of a magnetic field. Ions are created by electron-atom impacts. The ions are extracted from a small aperture in a high electric field, accelerated, then sometimes deaccelerated by a reverse electric potential gradient. (This "accel-decel" idea is often used to allow high ion currents to be extracted from sources at low power without being limited by space charge.) Ion current densities of 5-10mA/cm^2 are typical. For sources of positive ions (Cs, Hg, and Xe are the most developed for propulsion purposes) ionization fractions of 85-99.9% are achieved, with energy consumption of 80-800eV/ion produced. Electrodes are often eroded

[8] The fission cross section σ of ^{235}U roughly obeys $\sigma\sqrt{E}$ = constant = $97\sqrt{\text{eV}}$barns, where E is the neutron energy, for small E. (At least, this holds down to $E = 10^{-3}$eV [55].)

[9] A different interesting idea, which has not been exploited so far for space propulsion purposes, is the fact that sputtering solid surfaces with cesium ions will often yield sputtered negative ions. If these could be separated efficiently from the cesium, it might be possible to get a versatile source of negative ions.

by sputtering, but this problem seems to be eliminable by using liquid electrodes (e.g. Hg, for an Hg ion source). Even then, though, sputtering will still occur on non-electrode surfaces. Lifetimes of 1-3 years are typical. These sources are usually used to make positive ions, but if the extractor has its polarity reversed, they can also be used to make negative ions – although usually at low percentages and hence with the consumption of higher energy per ion. Usually only negative hydrogen ions are produced in this way. For example [1] describes a source of H^- ions consuming 9keV per ion produced and with "$> 5\%$" ionization fraction.

Positive ions of many metals, including multiply charged ones, also may be produced by spark discharges in vacuum [12].

4.4.2 Ions via charge transfer to/from surfaces

Positive ions may be produced [5][11][23][27][34][50][51] by passing gas atoms[10] with low ionization energy (Cs= 3.89eV, Rb= 4.18eV, K= 4.34eV, Ba= 5.12eV, Na= 5.14eV) past a high temperature (1350 Kelvin) metal surface with higher work function, such as tungsten ($\approx$ 5eV) or osmium (5.93eV). Porous (80% dense) tungsten electrodes made of sintered 4μm powder have been made for this purpose. Current densities of Cs^+ ions of 10-20mA/cm^2 have been achieved in this way with neutral fractions of .1-2%; the energy consumption is 500-700eV/ion. (Again the accel-decel technique is useful.) These ion sources have lifetimes of .5-3 years. "Poisons" which lower the work function of the tungsten surface can be cleaned off by occasionally heating it to higher temperature to try to boil them away.

Negative ions may be produced similarly by passing atoms of a gas with high electron affinity (such as iodine; all 4 halogens have electron affinities $>$ 3eV) past a high temperature (1800 Kelvin) metal surface with lower work function, such as GdB_6 (2.8eV) or LaB_6 (2.6eV). Current densities of I^- ions of 15mA/cm^2 were achieved in this way with GdB_6 (and it is believed that higher currents ought to be possible) at 10^{-5}torr of I gas pressure [21]. This source apparently was never developed beyond an experimental stage, and it operated for only 193 hours before fracturing. The ionization fraction was not measured, but presumably was high.

4.4.3 Ions via field-aided evaporation from liquids

Conductive liquid bodies exposed to an electric field can and will, under the influence of surface tension and electrical forces, develop conical points, or even cusps, on their surface [52][53][54]. In liquid metals (which are, after all, just positive ions in a sea of negative electron charge), the extremely high electric fields at these cusps or points will (if they are oriented outward) cause positive ions to be emitted [42][6][16][20]. This idea works with a large variety of metals (at least 15 have been tried [16] successfully), although metals with low vapor pressure (to avoid contamination of the ions by neutrals) and low melting point are preferred.

An ion source of this kind was developed [6] at the European Space Agency for space propulsion purposes. It seems best to expose the liquid metal to the electric field through long (1-10cm) narrow slits in a solid metal sheet. Liquid Cs has been preferred due to its low (70 mNewton/meter) surface tension (and hence excellent wetting properties), low melting point (30C), and low ionization energy. This kind of source can produce ion current densities of 2.5mA/cm^2, and with proper design parameters (for [6]'s liquid Cs source: 10mA and 1.1μm-wide, 8cm long slits) will have ionization fractions of 80%. The energy efficiency is excellent – approaching theoretical limits.

This method inherently only seems capable of producing positive ions, since liquid metals are mixtures of positive ions and electrons, and since [42] ions already in the liquid are extracted from it by the field (as opposed to neutral atoms being ionized by the field). Any attempt to produce negative ions from liquid metals would simply eject bare electrons.

However, I would like to suggest that this idea also *can* be used to produce negative ions by using *molten salts* instead of liquid metals. (This is a new idea.) Molten salts [8][9][35][40] are mixtures of negative and positive ions without free electrons (indeed, they are usually transparent to light). They typically have comparable viscosities, surface tensions, and densities to liquid metals, but their thermal conductivities are typically $\approx$ 150 times smaller and their electrical conductivities $\approx$ 15000 times smaller. Note, the electrical conductivities of molten salts are still large, despite this. For example, molten NaCl at 807C has a conductivity of 3.6mho/cm, which is 2950 times smaller than the conductivity of liquid Hg at 0C, but still is 3600 times larger than the conductivity of a room temperature aqueous solution of NaCl. This conductivity is essentially infinite, as far as a microscopically sharp point on the liquid's surface is concerned.

Indeed, molten salts may be more versatile ion sources than liquid metals since they should allow the production of more complicated ions, such as (perhaps) sulfate (SO_4^{--}), semisulfate (MSO_4^- where M is a metal), tungstate (WO_4^{--}), and complex organic ions. Eutectic salt mixtures[11] can feature low melting points, for example $AlCl_3$:NaCl:KCl in a 127:40:33 molar ratio melts at only 89C (and the quaternary eutectic with LiCl added, at only 61C). Some other eutectics include LiCl:KCl (58:42) melting at 325C (as opposed to pure LiCl or pure KCl at 610C and 772C respectively); $AlBr_3$:RbBr melting at 83C ($AlBr_3$: 97.5C; RbBr: 680C); LiBr:KBr (60:40) melting at 348C (LiBr: 550C; KBr: 734C); $ZnSO_4$:Na_2SO_4 (55:45), melting at 472C. For making low melting iodide eutectics, AlI_3 (melts 191C, boils 386C) is probably useful, although all compositions involving AlX_3, $X\in\{Cl,Br,I\}$, may exhibit high vapor pressure.

Of course, as Cs^+ ions are extracted from liquid Cs, or

[10]Conceivably, some *molecules* with low ionization energies might also work, e.g. K_2 with 4.04eV and UO_2 at $5.4 \pm .1$eV.

[11]Eutectic metal mixtures, such as lead-tin (37:63) solder, can also feature low melting points (183C), but would not be recommended for our purposes since they would produce ion mixtures. In contrast, with the molten salt eutectic mixtures listed, there is a *unique* kind of negative ion.

Cl^- ions from NaCl melt, etc., these elements need to be replenished and resupplied to the melt.

Note, I anticipate that the molten salt cusp source proposed here should be more versatile and energy efficient than all other known kinds of negative ion sources. Hence it should have many uses besides for space propulsion, e.g. accelerators, fusion research, ion microprobe, ion implanters, ion milling, mass spectrometers, and isotope separation.

It is also possible to produce $\pm$ charged solid or liquid particles containing many atoms (for example gold colloid "smoke," or glycerol droplets doped with H_2SO_4) but these inherently have mass to charge ratios $\geq 10000\text{amu}/q_e$ and hence seem uninteresting for our present purposes.

4.4.4 Lifetimes

Unfortunately, none of these kinds of ion sources have very long lifetimes. (Many have been certified for 10000 hour operation in space, but that is the longest I've heard of.) This level of reliability (a few years) might be adequate for some alpha electric rockets, but needs to be considerably improved before it can be used in nuclear electric (or any) rockets with thrust times of ≥ 100 years.

4.5 Ion accelerator

There is only one kind of particle accelerator which seems to come close to our current and voltage demands (§4.1): the Cockcroft-Walton accelerator [17][36][43]. This consists of a voltage multiplier circuit, made of capacitors and diodes, to convert comparatively low voltage AC into higher voltage DC. Devices of this nature have been constructed capable of supplying 2.5MV at 200mA, which exceeds our requirements (§4.1). Unfortunately, that particular generator, which was air-insulated [43], was 16.4meters high. It is possible to shrink the dimensions by up to a factor of k by using an insulator k times better (in terms of breakdown strength) than air, e.g. pressurized heavy gases, or oil. (Oils can be > 50 times better insulators than air.) Thus, the same manufacturer made a 1.0MV, 120mA supply fitting in a tank of pressurized SF_6 only about 3-4 meters long and about 2meters in diameter. Even these reduced-size devices are large and massive (or always have been, so far), for example the tank (for holding the insulating gas) *alone* for the 3MV, 15mA supply of [48] weighed 18 tons. It was found convenient [43] to generate the "low" (actually, of order 100kV) voltage AC using an iron-core transformer operating at 2kHz.

Then, within a large metal sphere charged to this voltage by the power supply (the sphere in [48] weighed about 1 ton), we place an ion source; the ions are accelerated by the potential gradient through an evacuated tube leading out of the sphere.

Let us quickly dismiss the main alternatives to Cockcroft-Walton. Van de Graaff accelerators generate too little current. Their current is limited by the density of charge which can be carried on the moving belt (limited by air breakdown, in practice, to about 1.3nanoCoulomb/cm^2) and by the velocity of that belt (20meter/second). N.J.Felici developed a version of the Van de Graaff generator in which the belt was replaced by a rigid rotor (this allowed higher speeds and better reliability – belts had to be replaced after < 3000 hours of running), and by using high pressure gas insulation, higher charge densities may be placed on the belt or rotor before breakdown. But even with these ideas, nobody has been able to attain more than a few mA.

Linear accelerators and cyclotrons are, even with superconducting magnets (which decrease the mass and the power requirements greatly), tremendously large and massive. For example, the Chalk River 10MeV superconducting cyclotron [32] has diameter and height about 3meters, uses a 5Tesla field, and weighs about 250 tons.

So far, I am unaware of any Cockcroft-Walton accelerator with oil or vacuum insulation (in terrestrial applications, these are regarded respectively as too messy and impractical) but these are the obvious choices in space. Let us assume highly optimistically[12] that a 1MV Cockcroft-Walton device could be built, with oil insulation, less than 1 meter in diameter and massing only 200 kg.

4.6 Energy convertor

4.6.1 Thermoelectric

Thermoelectric convertors based on, e.g., lead telluride, have the capability to convert heat directly into electricity. Since they have no moving parts, they can be highly reliable. Unfortunately systems based on such convertors have, so far, 5-6% total system efficiency at best, and (what really hurts) have power/mass ratios of at best 26 watts per kg. Hence to generate 50 kilowatts we would need 1900kg of thermoelectric elements.

4.6.2 Turboelectric

Therefore, we are forced to consider the more usual kind of energy convertor: heat is converted to mechanical energy by some sort of rotating machinery (e.g. a steam turbine), and then converted to electricity using a generator.

I have extreme reservations about the ability of such a convertor to keep working without maintenance, for ≥ 100 years. I've never heard of the achievement of such reliability. Nevertheless (in order to proceed, as opposed to giving up immediately) we shall assume it is possible.

The questions that then matter are: how efficient will it be, and how much will it mass?

Commercial nuclear power plants achieve efficiencies of 30%.

Mass estimates may be provided by considering the masses of commercially available engine/generator sets.

The Kohler model 65/1PF1P marine generator set (including 4 cylinder engine) generates 65 kilowatts. It weighs 992.5kg. That is 65.5watts/kg. (It comes with a 5 year limited warranty including 1 year unlimited hours warranty.)

[12] Since my main purpose it to prove the *inferiority* of the nuclear-electric plan, optimistic assumptions like this one only help me.

If we are interested just in heat→mechanical energy conversion, good estimates are available by considering aircraft engines.

The Hirth F-30EP 2-cycle aircraft engine generates 110hp at 6200RPM and is billed as having "one of the highest power to weight ratios available." It weighs 105lb including electric starter and exhaust, but without a gearbox (extra 24lb). Fan cooling adds 8.5lb. If we assume 113.5lb, this is 1594watts/kg. It is rated for 1000 hours at 75% power.

Indeed, almost all aircraft engines below 1megawatt generate 1000-2000watts/kg, a range which has remained unchanged for about 60 years despite great effort to improve it.

4.6.3 Obscurer types of energy convertors

Instead of thermoelectric elements, one could use "thermionic diodes" (electrons are boiled off a low work function surface into a vacuum, then extracted by another electrode). These have similarly poor mass/power ratios and efficiencies as thermoelectric devices and hence we must also exclude them.

Another idea is to avoid the need for solid moving parts, and also avoid the need for a low→high voltage convertor, by using an "electrogasdynamic" generator to generate high voltages directly. A jet of high speed gas is created by thermal means, which transports charged liquid droplets to a large spherical high voltage electrode. The physics of such generators are very interesting, but suffice it to say that they seem inherently incapable of generating sufficiently high currents for our purposes. The currents are limited by the transport speeds (roughly sonic, at most) and by the fact that trying to transport too high a charge density will cause the jet to widen (due to coulombic repulsion forces) by too much (i.e. wider than the diameter of the sphere).

4.6.4 Optimistic conclusion

So, let us suppose, very optimistically, that it is possible to build a sufficiently reliable energy convertor generating 2000watts of output power per kg, and running at 30% efficiency.

4.6.5 Pessimistic conclusion

It might be more realistic to assume half the efficiency, 10 times the mass, and still insufficient reliability.

4.7 Whole system analysis

Optimistic: Assume

1. 200kg of expellable mass,
2. an energy conversion device outputting 50kilowatts of electricity, massing 25kg and having 30% efficiency,
3. a 25kg nuclear reactor running at 620Kelvin and generating 167kilowatts of heat,
4. A 2-sided radiator, of area 364meter^2 (counting both sides) and mass 490kg, for radiating that heat away at 300Kelvin
5. A 1-megavolt accelerator and ion source capable of 50mA of beam current (i.e., 100% efficient), and massing 260kg.

Result: $E = 1.47 \times 10^{14}$Joules, $M = 1000$kg, and $m = 800$kg. Exhaust velocity $v = \sqrt{2E/(M-m)} =$ 857km/second. Final rocket velocity $V = v\ln(M/m) =$ 191km/second$= 0.0006c$.

Pessimistic: Assume the accelerator masses 310kg and is only 50% efficient (but still produces a 1 megavolt, 50mA beam). Assume the efficiency of the energy convertor is only 15% (half as much) and its mass is 500kg (10 times worse power/mass ratio). The radiator will now mass 1960kg. Assume the nuclear reactor is 30kg. Result: $E = 1.47 \times 10^{14}$Joules, $M = 3000$kg, and $m = 2800$kg. The exhaust velocity $v = \sqrt{2E/(M-m)} = 857$km/second is the same. Final rocket velocity $V = v\ln(M/m) =$ 59km/second$= 0.0002c$.

In either the optimistic or pessimistic case, nuclear-electric performance is far worse than alpha-electric, as claimed.

4.7.1 Low power, wait long time to reach final speed

If the nuclear reactor is run at very low power (e.g., 50watts), and the particle accelerator at correspondingly low currents, then the radiator and the energy convertor can both be tiny, allowing much greater speeds due to reduced deadweight. Unfortunately, you'll now have to wait 10^5years to reach full speed.

Ultra-optimistically: assume the whole spaceship (nuclear reactor and accelerator) masses 20kg, except for the 200kg of expellable mass. Then $M = 220$kg, and $m = 20$kg. Final rocket velocity $V = v\ln(M/m) = 2800$km/second$=$ $0.0093c$.

More realistically: If the spaceship weighs $m = 220$kg, so $M = 420$kg, then the final rocket velocity is $V = v\ln(M/m) = 554$km/second$= 0.0018c$.

5 Acknowledgement

This whole endeavor was stimulated by remarks on nuclear-electric rockets made by Freeman J. Dyson. Apparently Dyson also discovered theorem 1, indeed before I did.

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