Orbital tether. Revised.

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Abstract — **Science fiction writers have proposed the idea of an "orbital tether;" a rope connecting the earth to an orbiting satellite. We examine the idea as a mathematical exercise and show that from the standpoints of mass, volume, energy, and strength of materials alone, there is no reason it would not work. The optimal shape of the tether may be found in closed form. We are *not* claiming it is presently economically or technologically feasible to build it. It isn't. All we are saying is that, if built, it would stay up for quite a while. The amount of time it would stay up seems to be limited to 10^4-10^5 years by radiation and meteoritic dust impact damage. Macroscopic meteors might have to be actively defended against, otherwise the cable might last only 3000 years. The uncertainty in this last figure is very large. These lifetime limits are in principle very serious since it might take that long to build the cable, or for the cable to "pay for itself."**

Keywords — orbital tether, uniform stress, optimal shape, form, structure, science fiction

1 Orbital tether

Several science fiction writers have used the hypothetical idea of an "orbital tether." This is a rope attached at one end to the ground and at the other end to a mass in geosynchronous (or actually, higher) orbit. The idea is that if such a thing existed, then space travel would be cheap and simple. To get into space, one simply hauls oneself up the rope. Were the rope to extend a factor of $\approx 2^{1/3}$ further than the radius (42139km) of a geosynchronous orbit, that is extending outward to radius 53092km, then in fact the end of the rope would be moving faster than escape velocity and one could get a "free ride" beyond geosynchronous orbit into outer space. Indeed, the amount of energy required to pull oneself up to geosynchronous orbit would be more than balanced by the amount of energy one could extract while being pulled outward by centrifugal force during this second phase of one's journey!

For those readers who cannot understand how it is possible to get a "free lunch" in this way... the energy is of course being provided by extracting angular momentum from the rotating earth.

The question is whether existing materials are strong enough that a tether made of them would hold together. We will see that the answer is YES, if the tether is optimally shaped and if it is constructed of an extremely strong and light material.

After I'd written this, I found out by reading the afterword in A.C. Clarke's novel *Fountains of Paradise* (Harcourt 1978) that the tether idea was apparently due to "a Leningrad engineer" named Y.N.Arsutunov (*Kosmolovskaya Pravda* 31 July 1960). It was later analysed by J.D. Isaacs, A.C. Vine, H. Bradner, and G.E. Backus: *Satellite elongation into a true "skyhook,"* Science 151 (1966) 682-683; and finally by J. Pearson: *The orbital tower – a spacecraft launcher using the Earth's rotational energy,* Acta Astronautica 2 (1975) 785-799. Pearson proposed some interesting uses for the tether including absorbing the energy of incoming spacecraft as well as for launching outgoing ones, and radioactive waste disposal.

All of these people were unaware of each other, and all of them found the same optimal shape I did. I am apparently the first person to examine the damping of vibrations in the cable, although Pearson did a computer analysis of resonance in an (assumed undamped) cable[1]. I have two big quibbles with the previous authors. First, they lacked care regarding materials strengths. Since the cost of the tether is exponentially sensitive to tensile strength, it is essential to be careful; a small overstatement can go a long way to making the tether look much less costly. (Doubling the strength will square-root the cost!) However, the overstatements made by Isaacs et al. and Pearson were not small. Isaacs et al. thought the best cable would be diamond, and used as their strength estimate a figure 5.6 times as large as ours. This was based on an incorrect value of the modulus combined with an unrealistic multiplier. Isaacs also overstated the strengths of graphite and silica by factors of 1.2-1.7 and 1.8-15 respectively.

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[1]Pearson's analysis is more sophisticated and complicated than mine, but I do not believe it is better. For example it is not correct to treat the system as linear with a driving force depending only on time... the self-limiting nature of the oscillations and the magnitude of the damping seem to me to be the most important features of the system and were both ignored by Pearson's computer.

Pearson thought the best material was graphite whisker and used as his "design point" a strength-to-weight ratio 2.4 times larger than the actual figure, based on some nowhere-cited "laboratory measurement." Also, all these sources pretended a cable would be as strong as the fibers it was made of, while actual figures for aligned composites (such as filament-wound fiberglass) show a strength reduction of typically a factor of 2. I appear to be the first author to be careful about materials strengths and who has looked up the primary sources for the strength figures.

What are often called "theoretical strengths" of materials are well known to have little or nothing to do with reality, since the theoretical model they are based on have little or nothing to do with the way substances actually fail under tension. It is unfortunate that the previous authors have often used such figures. They should really be regarded as very weak *upper bounds* on true strength, not estimates of it.

Second, the previous authors did not analyse radiation and meteor impact damage in the cable. Impacts capable of severely damaging the cable may occur about once every 3000 years, although the uncertaintly in this figure is very large. If such impacts were prevented, radiation and micrometeor damage would still limit the cable's lifetime to 10^4-10^5 years. Even the upper figure is cause for great concern, considering the cable might take this long to build.

2 Optimal shape, and some numbers

First, let us derive the optimal shape of the tether. By "shape" we mean the cross-sectional area $A(r)$ of the tether, as a function of the distance r from the Earth's center. By "optimal shape" we mean a shape such that the stress is uniform. In operation, every part of the cable will be stressed to just below the ultimate strength T of the material. Let the density of the material the cable is made of be ρ, let the surface gravity of the earth be g, let the equatorial radius of the earth be R_e, and let the angular velocity of the earth be ω.

The tensile force at r is $TA(r)$ (if the stress is near-ultimate) and is also $\int_0^r \rho A(r)(gR_e^2/r^2 - \omega^2 r)dr$. This leads to the differential equation

$$T\frac{dA}{A} = (\frac{\rho R_e^2 g}{r^2} - \rho\omega^2 r)dr$$

which may be integrated to yield the solution

$$A(r) = \exp(-\frac{\rho R_e^2 g}{Tr} - \frac{\rho\omega^2}{2T}r^2)C \tag{1}$$

where C is a constant. Interestingly, this solution continues to work even if the cable extends infinitely far, although its cross section would have to become exponentially small.

The "levitation condition," stating that the cable will stay up since the total (gravitational and centrifugal) force on it is upward, is

$$\int_{R_e}^{R} \rho A(r)(gR_e^2/r^2 - \omega^2 r)dr < 0.$$

We have

$$\int_{R_e}^{R} \rho A(r)(gR_e^2/r^2 - \omega^2 r)dr = \frac{Y}{\rho}[A(R) - A(R_e)]$$

so that if the cable is long enough so that its cross-section $A(R)$ at its far end is smaller than its cross section $A(R_e)$ at the Earth's surface, the levitation condition will be satisfied. In particular, the hypothetical infinitely long cable would stay up, indeed even if it were truncated at the "escape radius" of 53091 km. Of course, if one does not wish to make an infinitely long cable, an entirely equivalent effect may be gotten by attaching an appropriately-sized mass to the free end of the cable to simulate the presence of the "rest of it."

Second, let us consider the required strength of the material. The important dimensionless parameter $T/(\rho g R_e)$ is the maximum length, measured in earth radii, of a (uniformly thick) cable that could be held up by itself in a uniform gravity field of strength g.

In the below we will assume we have a material for which this is 0.07840, i.e. that can hold up $L = 500$ kilometers of itself. Graphite whiskers have $L \approx 900$, so there is some elbow room here to put in some glue between the fibers or to include a safety factor. The only other materials I currently know of that exceed the $L = 500$ kilometer bound are whiskers of B_4C, BeO, SiC, or sapphire and (probably) diamond, see the data tables in §6.

Examining (EQ 1), we see that the optimally-shaped cable is thickest exactly at geosynchronous orbit and becomes thinner at either greater or lesser larger values of r. The ratio (remember – cross section behaves as the width squared!) of this maximal width to the cable width at the Earth's surface is 126. The ratio of the cable width at "escape radius" 53091km to the cable thickness at Earth's surface, is 0.96. The payload capacity of the cable, i.e. the maximum weight (where "weight" includes acceleration) that could climb it, is the same as the tension in the cable for the part of the cable *below* geosynchronous orbit. (Weights clinging to the cable *above* geosynchronous orbit tend to increase rather than decrease the stress on the cable and would therefore be limited by the amount of *extra* strength the cable had left over, i.e. perhaps to 10% of the cable tension. On the other hand, larger weights could still slide freely along the cable.) Assuming the cable was made of stuff with similar density to graphite, $T \approx 10^{10}$Pa. In that case the payload capacity would be 3×10^7 Kg masses. If the weights were rising (or falling) quickly along the cable, coriolis forces would be introduced traverse to the cable. These would be less than 1 part in 500 of the weight (a static traverse force of this magnitude applied to the middle of the cable would of course cause the cable to bend by 1 part in 1000), if the rise/fall velocity was less than 280m/sec= 1000Km/hr. (= 70 hours to get from here to geosychronous orbit.)

If the material were only half as strong – strong enough to hold up only 250 kilometers of itself – then these thickness ratios would *square*, becoming 15761 and 0.92 respectively. By using a weaker but lighter material [e.g. linear polyethylene] on the portions of the cable with less tension but more gravitational/centrifugal acceleration, and a stronger but heavier material [e.g. sapphire whisker] on the portions of the cable near geosynchronous orbit with higher tension but less acceleration, the situation would improve. (All my analysis had assumed that the composition of the cable is homogeneous.)

Now let us make some remarks about the volume, mass, and energy embodied by the cable. If our cable has radius 10cm at the Earth's surface (hence maximum radius 12.6m) and density of order 2250 Kg/m^3, it would have volume of the order of 10^{10}m^3, mass of the order of 10^{14}Kg, and energy (gravitational and kinetic) of order 10^{21}J. This mass is the equivalent of a mountain, and this energy is equivalent to a consumption of about 7000 watts for each of the 5×10^9 humans on the earth, for one year, or to the total solar flux incident on the earth's outer atmosphere in about 2 hours. A much smaller version of the cable, with radius only 0.1mm at the Earth's surface, would mass only 10^{10}Kg (which is 5000 times the mass of the fully-fueled space shuttle) and could conceivably be used to haul up raw materials to use in thickening the cable – although since its cargo capacity would be 10^4 times smaller than the full-size cable (only 3000 Kg) it might then take 5000 years to haul up enough material to increase the cross sectional area by a factor of 2, even at hauling speeds of 1000km/hr.

3 Wind, fatigue, resonance, and radiation/meteor damage

Some people have expressed concern about forces due to winds in the atmosphere. Assuming a 200km/hr wind[2], and constant 1Kg/m^3 air density for a 10 km high length of cable (very conservative) and with a cable diameter of 20cm and assuming a drag coefficient of 0.5, we have that the total traverse force on the cable would be 3×10^6N. This is $< 1\%$ of the tension in the cable, but this effect (ratio of side-force to tension) would grow X times worse for an X-times thinner cable, so it might be felt necessary to protect the lowest 20km of cable by enclosing it in a 20km high tower.

Some people have expressed concern that the tether might somehow be unstable. It is obvious that in any static sense, it is intrinsically stable. The cable is under tension. If you shorten it or bend it, it will try to stretch and straighten back out in order to minimize the gravitational and centrifugal force pseudopotential. If you lengthen it, it will contract due to elastic restoring force, for any material whose Young's modulus was greater than the applied tensional stress (and every material in §6 has much greater Young's modulus than its tensile strength). The only concern is thus that it might be unstable in a dynamic sense, e.g. resonance.

The main vibration-exciting forces are thermal expansion from the sun (<1 part in 1000) and tidal forces from the sun and moon (amounting to ≈ 1 part in 500 of the Earth's gravity at geosynchronous orbit). Thus even if there were no damping whatever, any vibrations, either traverse or tensional, would tend to have a self-limiting amplitude of $\approx 0.4\%$ of the cable length.

The driving forces for tensional vibrations have periods of 12.03 and 12.4 hours. The sun and moon also drive *traverse* vibrations of the cable with periods of 24.06 and 24.8 hours.

Since the speed of sound in graphite fiber is about 15000m/sec, we see that the duration it takes for sound to make a round trip in a 53091km graphite cable (the period of the bass mode) would be about 2.0 hours – comfortably far below the driving periods.

The speed of traverse waves caused by "plucking the string" would be $\sqrt{T/\rho}$ which leads to a characteristic period of 13.8 hours. Since this *is* rather annoyingly close to the driving periods (≈ 24 hours) care will be required.

Stretching a string decreases its (configurational) entropy, thus forcing its (vibrational) entropy to increase. In other words, stretching a string will cause it to become hotter. If the stretching is done so slowly that the heat has time to escape (i.e. isothermal conditions), then there will be an irreversible loss of energy during the stretching process. This leads to damping of vibrations. We now analyse this in more detail.

A strain S in the cable (a vibrational perturbation small in comparison with the normal amount of strain) represents an elastic energy of STV, where V is the volume of the cable and T is the tensional stress the cable is under. Meanwhile thermodynamics tells us that the unavoidable energy loss caused by imposing the stretch S under

[2] As Pearson observed, the jet stream is not present at equatorial latitudes

isothermal conditions is $([1-2\sigma]C\rho VS)/(3\alpha)$ where σ, $0<\sigma<1/2$, is the Poisson ratio of the cable material, C is its specific heat capacity (i.e. C is in units of energy per temperature per unit mass), ρ is its density, and α is its coefficient of linear expansion (so that 3α is the coefficient of volume expansion).

Thus the energy of a vibration, divided by the energy loss per cycle (which is an estimate of the "quality factor" Q of the resonance) is at most $\frac{3T\alpha}{(1-2\sigma)C\rho}$. For our graphite[3] cable, we have $T = 10^{10}$Pa, $\sigma \approx .3$, $\rho = 2250\mathrm{Kg/m}^3$, $C = 711\mathrm{J/(KgK)}$, and $\alpha = 7.8\times 10^{-6}\mathrm{K}^{-1}$. Hence $Q \approx 0.37$.

Lest the reader complain that a bell is certainly not damped at anywhere near this rate... I claim the reason is that the high frequency vibrations of small objects are adiabatic, not isothermal. The cable has hours to cool off, a bell has submilliseconds.

Are conditions really isothermal? Well, the characteristic time for heat to conduct out of the center of a cable of radius R is $\tau = \rho C R^2/\kappa$. The value of R ranges from 10cm to 12.6m, and for graphite, $\kappa = 165\mathrm{W/(m\ K)}$, so we have τ ranging from 1.6 minutes to 430 hours. This is to be compared with the 12-24 hour periods of the driving forces, so conditions are very isothermal in the thin part, but not very isothermal in the thick part of the cable[4], but not terribly adiabatic either. If Q is multiplied by a factor of 10 to compensate for this lack of isothermality, we still have $Q \approx 3.7$ which isn't bad. By installing a system in the thick part of the cable designed to increase the rate of heat conduction, essentially isothermal conditions could be obtained everywhere without much penalty. $Q<1$ should be easily achievable.

To conclude our crude analysis of resonance: due to the self-limiting amplitude, the off-tuning, and this damping, resonance should not be an insurmountable problem.

Despite the small amplitude of vibrations, one might still wonder whether, after many cycles, "fatigue" might cause a failure. Some substances have "fatigue limits;" cyclic stress below the fatigue limit will never cause failure. ("Never?" Experimenters have tried 10^8 cycles.) Steels typically have a fatigue limit of $\min(T/2, 700\mathrm{MPa})$, where T is the tensile strength of the steel. Epoxy has a fatigue limit of ≈ 20MPa. On the other hand, some substances apparently have zero fatigue limits, and, if cyclically stressed, will *always* eventually fail. These include nylon and some aluminum alloys (although some other aluminum alloys have fatigue limits of 20-160MPa).

The fatigue properties of most superstrong materials are apparently unknown. But we are only asking for a fatigue limit of order 10^{-3} times the tensile strength, which is not a heavy demand. I suspect that most of the supermaterials will have nonzero fatigue limits satisfying this demand. In particular, one epoxy graphite composite appears to have a fatigue limit corresponding to a strain of 0.0046, which is about 20% of fracture strain (Ryder&Walker 1977).

Another uncertainty at present is how long the material of the cable would maintain its strength under continuous exposure to atmospheric and outer space conditions. Both fatigue and radiation-induced strength decay are easily studied experimentally, but I am unaware of any such studies for the superstrong materials in §6.

Let us assume the cable is protected from sunlight, the atmosphere, and radiation by a 1mm-thick Al coating. (One also might want to run some current through the coating as necessary to de-ice it.) Assume the coating provides complete protection against UV and higher wavelength photons and also from protons and α-particles, but no protection against X-ray and lower wavelength photons, nor against MeV electrons. Also assume that 10^4 Grays (a Gray is 1 joule of absorbed radiation energy per kilogram) will be enough to significantly reduce the strength of the cable. Humans, among the most radiosensitive organisms, die at a 10 gray dose. Fruit flies can survive 10^3 Grays. Certainly virtually any material would experience extreme radiation damage after 10^7 Grays, since that would be about equal to the chemical self-energy of the material! Since strength is determined by the weakest weakness, 1000 times less Grays than the chemical self-energy of tha cable would seem to be enough to cause severe strength reduction. Radiation damage in metals is self-healing at reasonably warm temperatures, but most insulators do not self-heal (for example, 10^9-year old radiation tracks are visible in moon rocks) and all the good cable materials in §6 are nonmetals. The flux of electrons with energies >1.6 MeV in the most intense regions of the outer Van Allen belt is $10^8/\mathrm{m}^2/\mathrm{sec}$. For a 10cm-radius graphite cable, this would be about 7 Grays per year. (The effect is X times more severe for an X times thinner cable; recall that 10cm was the thinnest part of our cable.) Electrons with energies of a few MeV are absorbed mostly by the outer 1cm of cable so they may not be too destructive. But the flux of ≥ 1GeV cosmic rays is about 1 per cm^2 per sec. GeV electrons will penetrate materials to substantial depth. That causes, for a cable 10cm in radius, 32 joules per year per meter to enter cable, leading to 0.45 grays per year, leading to a 22000 year lifetime before getting a 10^4-Gray dose. But only 1% of cosmic ray particles are electrons, so the lifetime is 2×10^5 years. So this leads to estimates for the cable lifetime of somewhere between 2×10^4 and 2×10^5 years for a 10cm-radius cable, depending on the shielding, the absorptivity of the cable, and the endurable dose.

This limitation seems to be extremely serious! In principle segments of the cable could be replaced periodically, but at this rate of strength decay, the replacement rate would be at least comparable to the payload capacity of the

[3] And apparently, under isothermal conditions every material in §6 would have $Q<3$, which is satisfactory damping.

[4] Much larger problems would arise if the cable were made of fused silica fiber, a tremendously good thermal insulator for which $\kappa = 1.37\mathrm{W/(m\ K)}$, $C \approx 900\mathrm{J/(KgK)}$, $\rho = 2200\mathrm{Kg/m}^3$, so that (where the overbar denotes length average) $\overline{\tau} \approx 1$ year. Sapphire might also be troublesome, but all other substances in §6 have $\overline{\tau} < 200$ hours. One easily verifies that the heat-flow bottleneck in thick cables is conduction and not blackbody radiation from the cable's surface.

cable. The "much-smaller version" of the cable, mentioned above, would almost certainly not last long enough to serve its intended role as a "bootstrap."

(Some References: W.N. Ness: *The radiation belt and magnetosphere*, Blaisdell Waltham MA 1968; O.C. Allkoper: *Introduction to cosmic radiation*, Verlag Karl Thiemig Munich 1975; D.S. Billington and J.H. Crawford: *Radiation damage in solids*, Princeton University Press 1961; Encyclopedia Brittanica articles on *Radiation* and *Solar system*. For below, see also S.G.Love&D.E. Brownlee: *A direct measurement of terrestial mass accretion rate of cosmic dust*, Science 262 (1993) 550-553 and E. Grün, H.A. Zook, H. Fechting, R.H. Giese: *Collisional balance of the meteoritic complex*, Icarus 62 (1985) 244-272.)

The analysis is similar if instead of radiation damage, we are concerned with damage caused by micrometeroid impact. Again, for a cable X times thinner, whose surface to volume ratio is X times larger, the problem is X times more serious, so we will concentrate on the thin part of the cable at the critical point in our analysis. It has been estimated based on studies of metal panels on satellites that 4×10^7 Kg of dust (particles 0.1-1mm in diameter, most commonly 0.2mm) hit the Earth per year. Assume the average particle has speed 17Km/sec. This dust flux (2×10^{-15}Kg/m^2/sec) represents an energy flux (3×10^{-7}J/m^2/sec) actually much smaller than the energy flux ($[1.6\text{-}25] \times 10^{-6}$J/m^2/sec) in cosmic rays and the Van Allen belts. Anyway, one estimates that ≈ 200 Kg of dust will hit the cable per year, which is $\approx 10^{10}$ particles, or about 4 impacts per square meter of cable surface per year. If again we assume that 10^4 Joules of energy need to be deposited per Kg of cable to kill it – except that we will now use a figure of 10^3 instead of 10^4 because dust impact causes more concentrated damage – it would take 10^5 years to kill the thin part of the cable.

On the other hand, a particularly big piece of "dust," say the size of a grapefruit, could wipe it out instantly. I do not know the size distribution of meteors so I cannot be certain of the probability of this. One estimate: the moon has 3×10^{12} craters of > 1m diameter. If they were all formed during the last 10^9 years, that would be a flux of 2.5×10^{-18} rocks/m^2/sec. Based on a cable surface area of 2.5×10^9 square meters, the cable can expect to wait only 5 years before acquiring such a crater! There are reasons to believe the cable is actually much safer than this: many craters on the moon are secondary impacts, many moon craters were formed in a previous rock-rich era and not during the last 10^9 years (evidence: the lunar seas have many fewer craters). Extrapolating a semi-empirical model for Grün et al. 3 orders of magnitude further than they were willing to go, one estimates the flux of 100 Kg rocks to be 5×10^{-22} rocks/m^2/sec. That would result in an expectation value of 25000 years between impacts. This may be an overestimate of the lifetime. The uncertainties are very large.

As a last resort, if one were afraid that the cable were going to break, one could try disintegrating the lowest 200km of the cable. The rest of the cable would then rise into orbit and the Earth's inhabitants would hopefully not be endangered.

4 Installation "plan"

The simplest way to install the tether might be to start with it wound up on reels located in an orbit of the appropriate height (near geosynchronous), and then gradually unwind the cable both inward and outward. Tidal forces would automatically orient the cable vertically. At the final stage of unwinding it would be necessary to catch the end of the cable and attach it to the Earth's surface, while simultaneously unwinding the far portion of the cable quickly enough to keep it levitated, but not so quickly that it would fly away before you caught it... I have not analysed this process. It might be best for the touchdown to be at sea.

5 Conclusion

From the standpoints of mass, volume, energy, and strength of materials alone there is nothing in principle to stop such a tether, once built, from staying up for a few centuries. Perhaps the biggest threat to its well being is catastrophic meteor impact; we can expect a serious impact once every 3000 (?) years. If the cable were defended from large impacts, radiation and micrometeor impact damage would still limit its lifetime to somewhere between 10^4 and 10^5 years. The fact that previous authors had overestimated materials strengths makes building a tether an even more daunting and less rewarding project than it would otherwise have been: The projected mass of a graphite whisker composite cable, of radius 10cm at the Earth's surface (swells to radius 12.6m at geosynchronous orbit) is 10^{14} Kg. Building the cable by "bootstrapping" (using a much smaller cable to haul material for a larger one) runs into the problem that X-times thinner cables will decay, due to radiation and micrometeors, at least X times faster.

6 Data and brief survey of superstrong materials

Equatorial Radius of earth $R_e = 6378533$m. Equatorial surface gravity of earth (if earth were not rotating) $g = 9.78$m/sec^2. Earth's rotation rate $\omega = 7.292118 \times 10^{-5}$radians/sec.

We now give a short table of the Density ρ ($\mathrm{Kg/m^3}$), Tensile strength T ($10^8\mathrm{N/m^2}$), and critical length L (the length of itself it will hold in km) of various strong materials.

Data taken from A.Kelly and N.H.MacMillan: *Strong Solids* (Oxford UP 1986 ISBN=0-19-851362-3), which we will call "KM," K.Friedrich: *Ultrahigh strength materials* (VDI-verlag 1984, ISBN=3-18-148205-6), Michael Bever (ed): *Encyclopedia of Materials Science*, MIT Press 1986, and miscellaneous journal papers (individually cited). Many data on commercially available materials will be found in the catalogue of the Goodfellow material company (Malvern, PA 19355). In cases in the table with no accompanying citation, the datum is usually well known and noncritical, and one may refer to usual engineering and physics handbooks. I thank Allan Schweitzer for helping to compile this data.

Silica fibers vary dramatically in strength depending on their surface condition, temperature, and the surrounding medium (wet or dry). They are strongest when very cold and very dry; the papers by Kurkjian et al. and Proctor, Whitney et al. found strength values ranging from 5.0GPa (mean, in air) to 15.2GPa (max, in liquid He) for undamaged fibers, but surface damage introduced during typical manufacturing procedures usually reduces strength by a factor of 300, according to Wiederhorn 1984, or 50, according to Bansall&Doremus 1986. Similar comments apply to commercial "S-glass" fiber, which is 65% SiO_2, 25% Al_2O_3, and 10% MgO, except that the typical strength reduction factor from typical surface damage is "only" ≈ 30, according to Bansall&Doremus 1986. We are taking the attitude that in principle such damage is avoidable, so we have given figures for undamaged fibers. On the other hand, silica fibers are known to suffer from "static fatigue," which can cause their strength to decrease (over a period of years) by a factor of 10.

The strength of beryllium varies according to the method of preparation; drawn wires are twice as strong as typical bulk beryllium.

Ultrastrong polyethylene fiber, in which the polymer fibers have been straightened and aligned during polymerization by viscous shearing, is 100 times stronger than bulk unaligned polyethylene. "Spectra B" commercial UHS polyethylene fiber is available from Allied Chemical with 3.1 GPa tensile strength and specific gravity 0.97 (1-3% denser than normal polyethylene). Strengths ranging from 2-4.6GPa, smoothly depending on diameter and draw ratios, were found by Smook 1984. (See also Keller&Barham 1981). Problems with this substance include creep under stress and a fatigue limit which seems to be zero.

There are many grades of commercial carbon fiber; the figures given are for Hercules Magnamite IM6 fiber (Hercules Inc. Magna Utah 84044), one of the strongest. The weakest grades are only 40% as strong as this.

Ultrastrong steels are over 10 times as strong as the usual "mild steel."

"Filament wound" or "aligned" composite materials seem to be about 50% as strong as the fibers inside them.

Whiskers grow slowly; probably < 1Kg of whiskers have ever been intentionally produced. Their lengths range from < 1 to 30mm and they are strongest when they have polished surfaces and are $\leq 5\mu$m in diameter. Diamond whiskers have never been produced, as far as I know, and their strength is a guess based on the known bulk modulus of diamond (443 GPa according to Liu&Cohen 1989) and the known ratios (23-28) of modulus to whisker tensile strength for other strong hard materials with diamond-like structure. Recently discovered graphite-like "nanotubes" (Iijima 1991) which have 1-30nm diameter and up to 1μm length, conceivably are even stronger than graphite whiskers and may also be easier to produce in quantity.

Finally, I point out that ultimate strength is by convention figured from the breaking force divided by the cross sectional area of the *unstressed* sample. Since this area shrinks when stress is applied, the actual tensile strength in terms of the actual area is in fact 5-20% larger than the tabulated numbers. Spider silk, which elongates 35% without breaking, is exceptional; presumably it has a "true" strength $\approx 80\%$ larger than the tabulated figure.

material	ρ	T	L	reference
Graphite whisker	2250	196	891	Bacon 1960
B_4C whisker	2520	210	852	Gatti et al. 1967
BeO whisker	3010	248	842	Newkirk&Smith 1965
SiC Whisker	3217	210	667	Mehan&Herzog 1970
Sapphire whisker	3960-4000	210-280	537-723	KM p.360 & Soltis 1965
Diamond whisker (hypothetical)	3520	177	514	see text
Linear Polyethylene fiber	970	30-45	316-474	see text
Silica fiber	2200	50-130	232-604	see text
Commercial "Nicalon" SiC fiber (Dow corp.)	2550	83	333	Yajima 1976
Commercial Carbon fiber	1730	43.8	259	see text
Kevlar49 (Dupont corp.)	1440-1450	28-36.2	197-257	
S-glass (fresh drawn dry) fibers	2480	31-46	128-190	Bever 1986
Asbestos (Chrysotile 3cm) fiber	2500	31	127	Aveston 1969
Spider dragline silk	1200	13.8	118	Denny 1988
Boron	2355	16-24	68-102	Goodfellow, Talley 1959
Beryllium wire (commercial)	1850	13	72	Fulap 1962
"Patented" steel wire 0.9C 0.4Mn 0.2Si	7900	42	54	KM
Tendon of desert locust	1200	6	51	Bennett-Clark 1975
Titanium alloy (13V 11Cr 3Al) wire	4600	17	38	Zwicker 1974
HS steel(.30C,.56Mn,4.3Ni,1.4Cr, heat trtd smpl)	7800	17.2	23	
99.95 Tungsten	19300	39	21	
Al(5.5Zn,2.5Mg,1.5Cu,.3Cr,.2Mn) alloy	2800	5.65	21	A.S.M. Metals handbook
A strong Mg Alloy	1800	3.0	17	
Polyester fiber	1500-2100	2.1-3.5	14-17	
ASTM Al alloy no.17ST	2800	4.1	15	
Nylon66	1140	.63-1.0	6-9	
Epoxy	1200	.83	7	

References for table

J. Aveston: *The mechanical properties of asbestos*, J. Mater. Sci. 4 (1969) 625-633. Chrysotile is the most common mineral form (90%).

R. Bacon: *Growth, structure, and properties of graphite whiskers*, J.Appl.Phys. 31 (1960) 283-290. Whiskers usually failed by shear at the gripper and not by tensile failure; the highest observed strengths, 2GPa, were therefore always from abnormally ribbon-shaped whiskers. A later chemical process (US Patent 3378345 by R.G. Bourdeau and F.E. Papalegis 1968) produced higher volumes of graphite whiskers but their observed strengths were 4-7 times weaker.

N.Bansall and R.H.Doremus: *Handbook of glass properties*, Academic Press 1986.

H.C. Bennett-Clark: *The energetics of the jump of the locust*, J. Exp. Biol. 63 (1975) 53-83

M.W.Denny: p241-272 in *Mechanical Properties of biological materials*, (eds. Vincent and Curry) Cambridge Univ. Press 1988.

R.J.Fulap: Mater. des Engng. 55 (1962) 10

A. Gatti, C. Mancuso, E. Feingold, R. Mehan: suppl no. 1 to J.Phys.Chem.Solids 28 (1967) 317.

Sumio Iijima: *Helical microtubules of graphitic carbon*, Nature 354 (1991) 56-58 (see also Ebbesen et al.: Chem.Phys.Lett. 209,1-2 (1993) 83-90.

A.Keller and L.J. Barham: *High modulus fibers*, Plastic and Rubber Intl 6,1 (1981) 19

C.R. Kurkjian et al.: Appl. Phys. Lett. 28 (1976) 588-590

Amy Y. Liu and Marvin L. Cohen: *Prediction of new low compressibility solids*, Science 245 (25 Aug 1989) 841-842

R.L.Mehan and J.A. Herzog: p157 in *Whisker Technology*, Editor A.P.Levitt, Wiley NY 1970.

H.W. Newkirk and D.K. Smith: Amer. Miner. 50 (1965) 22.

B.A. Proctor, I.Whitney et al.: Proc. Royal Soc. A297 (1967) 534, A282 (1964) 43 and Physics Chem. Glasses 5 (1964) 1.

J.T. Ryder and E.K. Walker in *Fatigue of filamentary materials*, ASTM STP#636 (1977).

J.Smook et.al.: J. Mater. Sci. 19 (1984) 19-43 and 1359-1373 and 3443-3450.

P.J.Soltis: Bull. Amer. Phys. Soc. 10 (1965) 163.

C.P.Talley: *Mechanical properties of glassy boron*, J.Appl.Phys. 30 (1959) 1114-1115

S.M.Wiederhorn: Ann. Rev. Matls. Sci. 14 (1984) 373-403

S. Yajima et al.: Bull. Amer. Ceram. Soc. 55 (1976) 1065. The figure $T = 83$ is the max of several samples.
U.Zwicker: *Titan und Titanlegierung*, Springer 1974.